

考試科目	普通物理(二)	所別	應用物理研究所 (8162) (8163)	考試時間	3月6日(文)第四節
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1. [20 points] **Short Answer Questions**

- (a) What emit electromagnetic radiation?  
 (A) steady electric currents  
 (B) rotating electric charges  
 (C) oscillating electric dipoles
- (b) Which has more energy, a photon of ultraviolet radiation or a photon of infrared radiation?
- (c) An x-ray photon is scattered by an electron. What happens to the frequency of the scattered photon relative to that of the incident photon?
- (d) What is a scanning tunneling microscope (STM)? Explain how it works.

2. [12 points] **Electrostatics**

A spherically symmetric charge distribution with total charge  $Q$  has charge density  $\rho \propto 1/r$  for  $r < R$  and  $\rho = 0$  otherwise. Find the electric field and the electric potential everywhere in space (for  $r \leq R$  and  $r > R$ ). Take the potential to be zero at  $r = \infty$ .

3. [12 points] **Maxwell's equations**

- (a) A physicist claims to have found magnetic field lines as shown in Fig. 1. Is this finding consistent with Maxwell's equations? Justify your answer.

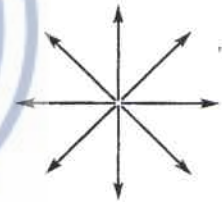


Fig. 1

- (b) Write the continuity equation expressing (local) charge conservation. Explain why the original form of Ampère's law, which does not include the displacement current term, contradicts the continuity equation.

4. [9 points] **Polarization of Light**

Consider two beams of light, (A) and (B), described as follows:

(A) a beam of fully linearly polarized light with the polarization direction making an angle of  $45^\circ$  with the vertical axis (the  $y$  axis); (B) an equal mixture of vertically polarized light (polarized in  $y$ ) and horizontally polarized light (polarized in  $x$ ).

How would you design an experiment to distinguish these two beams?

5. [10 points] **Interference**

Two waves of the same frequency  $\omega$  are described by

$$\mathbf{E}_1(\mathbf{r}, t) = \mathbf{E}_0 e^{i(\mathbf{k}_1 \cdot \mathbf{r} - \omega t + \phi_1)}$$

$$\mathbf{E}_2(\mathbf{r}, t) = \mathbf{E}_0 e^{i(\mathbf{k}_2 \cdot \mathbf{r} - \omega t + \phi_2)},$$

where the amplitude  $\mathbf{E}_0$  and the wave vectors  $\mathbf{k}_1, \mathbf{k}_2$  are constant real vectors. Determine the condition for constructive interference when the two waves are superposed.

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## 6. [15 points] Relativity

- (a) State the postulates upon which Einstein based his special theory of relativity.  
 (b) Maxwell's wave equation for the  $z$ -component of an electric field,  $E_z$ , propagating in the  $x$ -direction is

$$\frac{\partial^2 E_z(x,t)}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 E_z(x,t)}{\partial t^2} = 0,$$

where  $c$  is the speed of light. Is this equation invariant under a Galilean transformation to a reference frame moving with relative speed  $v$  in the  $x$  direction? Clearly show how you derive your answer.

## 7. [12 points] Quantum Physics

The figures below (Fig. 2) show normalized real wave functions at a certain time, plotted as  $\psi(x)$  versus  $x$ , for a quantum particle confined to the one-dimensional region  $-1 \leq x \leq 1$ :

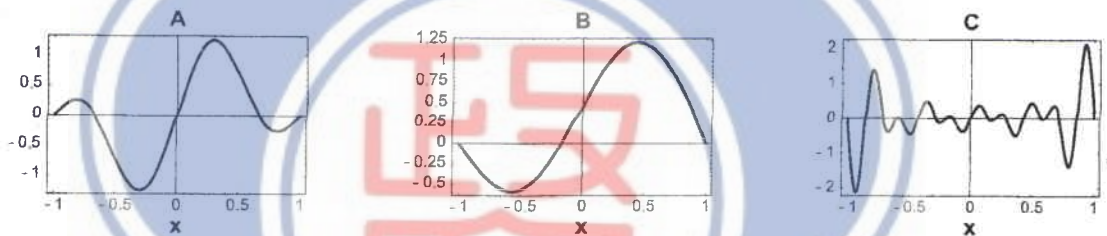


Fig. 2

- (a) Which wave function (A, B, or C) will yield the largest expectation value of the position  $\langle x \rangle$ ? Briefly give your reason.  
 (b) Which wave function (A, B, or C) will yield the largest value for  $\langle x^2 \rangle$ ? Briefly give your reason.

## 8. [10 points] Prerequisite Mathematics

- (a) Diagonalize the following matrix:

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

- (b) Consider the differential equation

$$\frac{d^2 f(x)}{dx^2} + k^2 f(x) = 0, \quad \text{with } k > 0,$$

which satisfies the boundary conditions:

$$f(0) = f(\pi) = 0.$$

Find the condition for  $k$  such that this boundary value problem has a non-zero solution.