國立成功大學 108 學年度碩士班招生考試試題

系　　所：電腦與通信工程研究所

考試科目：線性代數

考試日期：0224，節次：3

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (21 %) A denotes an $m \times n$ matrix. Make each statement True or False. Justify each answer. 3 % for each question. (每題 3 分)
   a. If $\boldsymbol{u}$ is a vector in a vector space V, then (-1)$\boldsymbol{u}$ is the same as the negative of $\boldsymbol{u}$.
   b. A subset H of a vector space V is a subspace of V if the following conditions are satisfied: (i) the zero vector of V is, and (ii) $\boldsymbol{u}$, $\boldsymbol{v}$, $\boldsymbol{u}+\boldsymbol{v}$ are in H.
   c. Nul A is the kernel of the mapping $\boldsymbol{x} \mapsto A\boldsymbol{x}$.
   d. The range of a linear transformation is a vector space.
   e. The set of all solutions of a homogeneous linear differential equation is the kernel of a linear transformation.
   f. If a finite set S of nonzero vectors spans a vector space V, then some subset of S is a basis for V.
   g. A basis is a linearly independent set that is as large as possible.

2. (20 %) Let $A = \begin{bmatrix} 1 & 0 & -2 \\ -2 & 1 & 6 \\ 3 & -2 & 5 \end{bmatrix}$, $\boldsymbol{b} = \begin{bmatrix} -1 \\ 7 \\ -4 \end{bmatrix}$, $\boldsymbol{c} = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}$, and define a transformation $T: \Re^3 \to \Re^3$ by $T(\boldsymbol{x}) = A\boldsymbol{x}$, so that $T(\boldsymbol{x}) = A\boldsymbol{x} = \begin{bmatrix} x_1 - 2x_3 \\ -2x_1 + x_2 + 6x_3 \\ 3x_1 - 2x_2 + 5x_3 \end{bmatrix}$.
   a. Find an $\boldsymbol{x}$ in $\Re^3$ whose image under T is $\boldsymbol{b}$. (10 %)
   b. Determine if $\boldsymbol{c}$ is in the range of the transformation T. (10 %)

3. (9 %) Find a basis for the kernel and image of T. $T: M_{2\times2} \to M_{2\times2}; T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a+b & b+c \\ c+d & d+a \end{bmatrix}$.

4. a. (10 %) Find the best least-squares line fit to the points (2, 1), (3, 1), (0, 0), and (-1, 1).
   b. (5 %) Find the least-squares error associated with the above least–squares solution.

5. (25 %) Let A be the matrix given by $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$ with rank $A = 1$.
   a. Compute A^+ (pseudoinverse, the inverse of reduced singular value decomposition of A). (10 %)
   b. Find a least square solution for $A\boldsymbol{x} = [1, 0, 1]^T$. (10 %)
   c. Find the orthogonal projection of $\boldsymbol{b} = [1, 0, 1]^T$ on Col A. (5 %)

6. (10 %) Let A be the matrix of the quadratic form $9x_1^2 + 7x_2^2 + 11x_3^2 - 8x_1x_2 + 8x_1x_3$. Find an orthogonal matrix $\boldsymbol{P}$ such that the change of variable $\boldsymbol{x} = \boldsymbol{P}\boldsymbol{y}$ transforms $\boldsymbol{x}^T A\boldsymbol{x}$ into a quadratic form with no cross-product term.