系　　所：環境醫學研究所

考試科目：微積分　　　　　　　　　　　　　　考試日期：0224，節次：3

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※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (25%) Evaluate the following limits:

   (a) $\lim\limits_{x \to 2} \frac{\sqrt{1+\sqrt{2+t}}-\sqrt{3}}{t-2}$.

   (b) $\lim\limits_{x \to +\infty} (\sqrt{x^2+1} - \sqrt{x^2-x})$.

   (c) $\lim\limits_{x \to +\infty} (1 - \frac{2}{x})^x$.

   (d) $\lim\limits_{x \to 0} \frac{1}{x^3} \int_0^x \frac{t^2}{t^4+1}\, dt$.

   (e) Does $\lim\limits_{(x,y) \to (0,0)} \frac{(x+y)^3}{x^3+y^3}$ exist ? why?

2. (20%) Find the following integrals:

   (a) $\int_{-3}^3 (|x| - 1)\, dx$.

   (b) $\int_{-1}^1 \frac{1}{x^2}\, dx$.

   (c) $\int \frac{1}{1+e^x}\, dx$.

   (d) $\int x \ln x\, dx$.

3. (10%) Using $\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x-\mu)^2}\, dx = 1$, find $\int_{-\infty}^{\infty} \frac{x}{\sqrt{2\pi}} e^{-\frac{1}{2}(x-\mu)^2}\, dx$.

4. (18%) Find the following derivatives:

   (a) Assume that the equation $xz + y \ln x - x^2 + 4 = 0$ defines x as a differentiable function of two independent variables y and z. Find $\frac{\partial x}{\partial z}$.

   (b) Assume that $\int_{y^2}^0 t^2\, dt + \int_0^x \cos t\, dt = 0$ and y is a differentiable function of x. Find $\frac{dy}{dx}$.

   (c) $y = (\sin x)^{4x^2}$, $\sin x > 0$. Find $\frac{dy}{dx}$.

5. Let $f(x,y) = e^{x^2+2y^2}$, $x, y \in \mathbb{R}$.

   (i) (6%) Find the critical points of f.

   (ii) (6%) Use the Second Derivative test to classify the nature of the critical points.

   (iii) (5%) Find the relative extremum of f if it exists.

6. (10%) Evaluate the iterated integral $\int_0^2 \int_{2y}^4 e^{x^2}\, dxdy$.