國立政治大學 99 學年度研究所博碩士班招生考試試題 第 1 頁，共 1 頁

考試科目	微積分	所別	應用數學系 8111, 8116	考試時間	3月6日(六)第一節

1. (16%) Prove or disprove (by a counter example): Let f be a real-valued differential odd function defined on $\mathbb{R}$. Let $g(x) = |f(x)|$. Suppose that $f(x_0) = 0$ for some $x_0 \in \mathbb{R}$. Then g is not differentiable at the point $x = x_0$.

2. (16%) Let $f(x, y, z) = \max\{x, y, z\}$ for all $(x, y, z) \in \mathbb{R}^3$. Is f a continuous function? Justify your answer.

3. Let f be a continuous real-valued function defined on $[a, b] \subset \mathbb{R}$. Suppose that f is differentiable on (a, b). Let

$$G(x) = \int_a^x f(t)\,dt.$$

   (a) (6%) Show that G is well-defined.

   (b) (10%) Show that $G'(x) = f(x)$.

4. (a) (5%) State the Stoke's theorem.

   (b) (15%) Let $\mathbb{F}(x, y, z) = (3y, -xz, yz^2)$ be a vector field on $\mathbb{R}^3$. Let S be portion of the surface $z = (x^2 + y^2)/2$ below the plane $z = 2$, with the curve C as the boundary. Verify Stoke's theorem.

5. (16%) Let f be a real-valued function on $\mathbb{R}$. Suppose that

$$f''(x) = (x-1)^{71}(x-2)^{52}(x-3)^{111}$$

   and $f'(0) < f'(1) < f'(5) < f'(3)$. Show that $5f'(0) + f(0) \le f(x) \le 5f'(3) + f(0)$, for all $x \in (0, 5)$.

6. (16%) Find all possible real differentiable function on $\mathbb{R}$ such that the derivative of the function is itself. Justify your answer.

備註	試題隨卷繳交