


注意：考試開始鈴響前，不得翻閱試題，  
並不得書寫、畫記、作答。

國立清華大學 108 學年度碩士班考試入學試題

系所班組別：經濟學系

考試科目(代碼)：微積分與統計(4603)

### — 作答注意事項 —

1. 請核對答案卷(卡)上之准考證號、科目名稱是否正確。
2. 作答中如有發現試題印刷不清，得舉手請監試人員處理，但不得要求解釋題意。
3. 考生限在答案卷上標記「由此開始作答」區內作答，且不可書寫姓名、准考證號或與作答無關之其他文字或符號。
4. 答案卷用盡不得要求加頁。
5. 答案卷可用任何書寫工具作答，惟為方便閱卷辨識，請儘量使用藍色或黑色書寫；答案卡限用 2B 鉛筆畫記；如畫記不清(含未依範例畫記)致光學閱讀機無法辨識答案者，其後果一律由考生自行負責。
6. 其他應考規則、違規處理及扣分方式，請自行詳閱准考證明上「國立清華大學試場規則及違規處理辦法」，無法因本試題封面作答注意事項中未列明而稱未知悉。

國立清華大學 108 學年度碩士班考試入學試題

系所班組別：經濟學系(0546)

考試科目（代碼）：微積分與統計(4603)

共 2 頁，第 1 頁 \*請在【答案卷、卡】作答

[Instructions: Please do all questions and show your work in details.]

1. [5 pts] If  $\text{cov}(X + Y, X - Y) = 12$ ,  $\text{var}(X + Y) = 20$ , and  $\text{var}(X - Y) = 16$ , obtain  $\text{corr}(X, Y)$ .

2. [5 pts] If the probability of a random variable  $X$  with space  $R_X = \{1, 2, 3, \dots, 12\}$  is given by:

$$f(x) = k(2x - 1),$$

then, what is the value of the constant  $k$ ?

3. [5 pts] Is the real valued function  $f: \mathcal{R} \rightarrow \mathcal{R}$  defined by:

$$f(x) = \begin{cases} 1 + |x| & \text{if } -1 < x < 1 \\ 0 & \text{otherwise,} \end{cases}$$

a probability density function for some random variable  $X$ ?

4. [10 pts] Let  $X$  and  $Y$  have the joint density function:

$$f(x, y) = \begin{cases} x + y & 0 < x, y < 1 \\ 0 & \text{otherwise.} \end{cases}$$

(a) What is  $E(XY)$ ?

(b) What is the covariance between  $X$  and  $Y$ ?

5. [25 pts] Consider a simple linear regression model:

$$Y_i = \alpha + \varepsilon_i, \tag{1}$$

where  $\varepsilon_i \sim \mathcal{N}(0, \sigma^2)$ .

(a) What is the independent variable in Model (1)?

(b) Please minimize the residual sum of squares to obtain the Ordinary Least Squares (OLS) estimator ( $\hat{\alpha}_{OLS}$ ) of  $\alpha$ .

(c) Is  $\hat{\alpha}_{OLS}$  an unbiased estimator of  $\alpha$ ?

(d) What is the distribution of  $\sqrt{n}(\hat{\alpha}_{OLS} - \alpha)$ ?

(e) Consider the null hypothesis  $H_0: \alpha = 5$ . The regression output will give you directly the test result for  $H_0: \alpha = 0$  from a statistic/econometric software. Can you suggest a regression run to yield the test result directly from the regression output?

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共 2 頁，第 2 頁 \*請在【答案卷、卡】作答

6. (10 points) Evaluate the following limits:

(a)

$$\lim_{x \rightarrow 1} \frac{\ln x}{x - 1}$$

(b)

$$\lim_{x \rightarrow \infty} \frac{x + 2}{\sqrt{x^2 + 4} x}$$

7. (10 points) Let  $y = f(x) \equiv x^{\frac{5}{3}} e^{x^2}$ , find  $\frac{d}{dx} f^{-1}(x)$  at  $x = 1$ .

8. (10 points) Considering  $f(x, y) \equiv x^2 - 3xy + y^3 - 7 = 0$ , evaluate  $y'(x)$  at  $(x_0, y_0) = (4, 3)$ .

9. (10 points) Prove that  $f(x) = \frac{1}{x}$  for  $x \in (0, 1]$  and  $f(x) = 0$  for  $x = 0$  is Riemann-integrable on  $[0, 1]$  or not.

10. (10 points) Evaluate  $\int_0^1 \frac{1}{\sqrt{x}} dx$ , or if not Riemann-integrable, not defined.