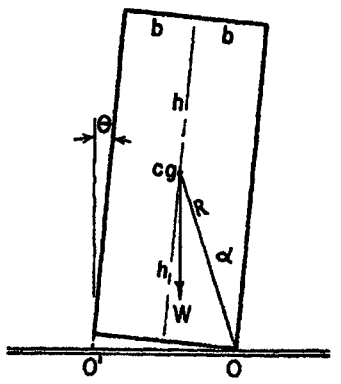
題號： 275　　國立臺灣大學 108 學年度碩士班招生考試試題

科目： 動力學(B)

節次： 7

1. (25%) The rigid block shown in the right figure will oscillate about the centers of rotation O and O' when it is set to rocking. The coefficient of friction is assumed to be sufficiently large so that there will be no sliding between the block and the base. The properties of the block are its weight W, its moment of inertia I_0 about the point O, and the location of gravity a distance h above the base and a distance b from the block side. The radial distance from the center of rotation O to the center of gravity is $R = \sqrt{h^2 + b^2}$. When the block is at rest, the line R makes angle α with the vertical. The tilting of the block from the vertical is measured by the angle θ.

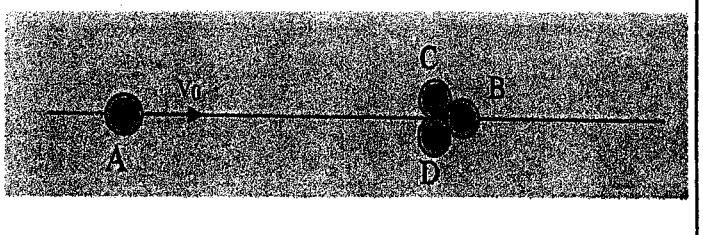


(i) Please find the equation of motion of the block. (10%)

(ii) If the block is tall and slender, find the reduced form of its equation of motion. (8%)

(iii) The block will tilt about O' after it falls back into the vertical position, and the impact is elastic. Find the time required to complete one cycle if the block is subject to the initial conditions $\theta = \theta_0$ and $\dot{\theta}$=0 at t=0. (7%)

2. (25%) On a horizontal frictionless plane shown in the right figure, a rigid ball A with an initial velocity $\mathbf{v_0}$ makes an elastic collision with three closed contacted rigid balls B, C, and D which are all at rest. The four balls are of the same mass m and the line connecting with the ball centers of C and D is perpendicular to the moving direction of ball A. What are the velocities of the four balls after the collision?

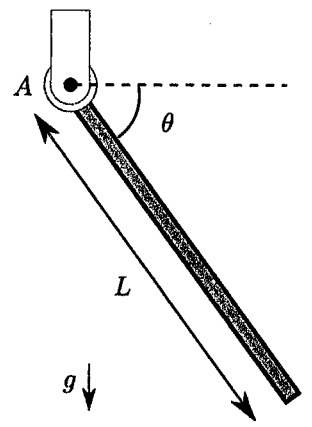


3. (30%) A ball of mass M is attached to a spring. The position of the ball is denoted by x and the acceleration is denoted by a(x). Please find the maximum position of the ball for the following cases.

(i) Assume the acceleration of the ball is given by $a(x) = -\alpha x$, where $\alpha = 1/s^2$. The ball has a velocity of $v = 2$ m/s when $x = -1$ m. (15%)

(ii) Assume the acceleration of the ball is given by $a(x) = -\alpha x - \beta x^3$, where $\alpha = 1/s^2$ and $\beta = 2/(m^2 \cdot s^2)$. The ball has a velocity of $v = 3\sqrt{2}$ m/s when $x = -1$ m. (15%)

4. (20%) The moment of inertia with respect to a given axis of a solid body with density ρ is defined by the volume integral $I = \int_V \rho r^2 dV$, where r is the perpendicular distance from the axis of rotation. A slender rod has a mass M and length L.



(i) Find the moment of inertia of the rod rotating about its center, I_{center}, and the moment of inertia of the rod rotating about its end, I_{end}. (10%)

(ii) Under the action of gravitational force, this rod is released from rest at $\theta = 0°$. Find the force $\vec{F}$ exerted on the rod by the pin A at $\theta = 90°$. (10%)

試題隨卷繳回