1. (20%) Find the general solution of the non-homogeneous differential equation

$$\left(D^2+6D+9\right)y = 16e^{-3x} \Big/ \left(x^2+1\right)$$

2. (20%) Find the Fourier integral of Delta function $\delta(t-a)$, and then using the result to evaluate the Fourier transform of $A\sin\beta t$. a and β are constants.

$$\left[\ \text{hint:} F(\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)\cdot e^{-i\omega t}dt,\ f(t)=\int_{-\infty}^{\infty} F(\omega)e^{i\omega t}d\omega\ \right]$$

3. (20%) Consider the following eigenvalues problem of $y(x)$:

$$y''+\lambda^2 y=0 \quad -p \geq x \leq p$$

$$B.C.\ \begin{cases} y(-p)=y(p) \\ y'(-p)=y'(p) \end{cases}$$

if λ_m and λ_n are two distinct eigenvalues of the problem, show that the corresponding eigenfunctions $y_m(x)$ and $y_n(x)$ are orthogonal in (-p, p).

4. (20%) Find the displacement of the semi-infinite string subject to the following equation and conditions

$$\text{Governing equation: } \frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2},\quad 0<x<\infty,\quad 0<t<\infty$$

$$BCs:\begin{cases} u(0,t)=\begin{cases}\sin 2t & \pi<t<3\pi \\ 0 & otherwise\end{cases} \\ \lim\limits_{x\to\infty} u(x,t)=0 \quad t\geq 0 \end{cases}$$

$$ICs.\ \begin{cases} u(x,0)=0 \\ u_t(x,0)=0 \end{cases}$$

5. (10%) Let λ be an eigenvalue of the Hermitian matrix **U**. Then prove that λ must be real.

6. (10%) Find the least square ("best") solution of the following system:

$$\begin{aligned} x_1-2x_2&=-2 \\ 2x_1+x_2&=3 \\ x_1+3x_2&=1 \\ 4x_1+x_2&=4 \end{aligned}$$

試題隨卷繳回