考試科目	微積分	系所別	風險管理與保險學系 精算科學組	考試時間	2月18日(一)第一節

Problem 1 (8 points) Evaluate the integral

$$\int_{-1}^{1} \frac{e^{2x}}{1+e^{x}} dx.$$

Problem 2 (8 points) Evaluate the improper integral

$$\int_{0}^{\infty} x^{n} e^{-x} dx,$$

n is a positive integer.

Problem 3 (10 points) Find

$$\lim_{n \to -\infty} (1+\frac{1}{x})^{x}.$$

Problem 4 (10 points) Find

$$\lim_{n \to \infty} (\frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{n+n}).$$

Problem 5 (14 points) Consider the cardioid given by $r = 1 - cos\theta, 0 \leq \theta \leq 2\pi$.

(a) Find the area enclosed by this curve.

(b) Find the length of this curve.

Problem 6 (10 points) Find

$$\int_{0}^{1} \int_{\sqrt{x}}^{1} sin(y^{3}) dy dx.$$

Problem 7 (16 points) Determine whether the series converges absolutely, or converges conditionally, or diverges.

(a)

$$\sum_{n=1}^{\infty} (-1)^{n} \frac{3 \cdot 5 \cdot 7 \cdots (2n+1)}{n! \cdot 3^{n}}.$$

(b)

$$\sum_{n=1}^{\infty} ln(1+\frac{1}{\sqrt{n}}).$$

Problem 8 (10 points) Let

$$f(x) = sin(x^{3}).$$

Find

$$f^{(15)}(0).$$

Problem 9 (14 points) Use the method of Lagrange multipliers to find the extreme values of z on the curve of intersection of

$$x^{2} + z^{2} = 1$$

and

$$y^{2} + z^{2} + z = 1.$$

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。