考 試 科 目	數理統計學	系 所 別	統計學系	考 試 時 間	2 月 18 日(一) 第三節

1. A system has 4 components. It works if both component 1 and at least 2 of the other components operate, otherwise it fails.   Let Y be the lifetime of the system, X_i be the lifetime of the ith component, and X_i's are *i.i.d.* with a continuous distribution function $F(x)$.
   Derive the distribution function and the pdf of Y.   (10%)

2. Let $X_1, \cdots, X_n$ be a random sample from $f(x) = \theta \eta^\theta x^{-(\theta+1)}, \ x \geq \eta; \ \theta, \eta > 0$.
   (1) Find the MLEs (maximum likelihood estimators) of θ and η.   (8%)
   (2) Suppose $\eta = 1$, then the pdf becomes $f(x) = \theta x^{-(\theta+1)}, \ x \geq 1; \ \theta > 0$.
     (a) Find MVUE (minimum variance unbiased estimator) of $\theta + \theta^2$.   (12%)
     (b) Find an approximate $100(1-\alpha)$% confidence interval for θ.   (8%)
     (c) Let $X_{(1)}$ be the smallest order statistic.   (12%)
       (c1) Is it true that $\ln(X_{(1)}) \xrightarrow{\ P\ } 0$?   Show your work.
       (c2) Find the limiting distribution for $n(X_{(1)} - 1)$.

3. The joint distribution of X and Y is $f(x,y) = \begin{cases} 8xy, & 0 \leq x \leq y \leq 1 \\ 0, & \text{elsewhere} \end{cases}$.
   (1) Suppose $Y = 0.6$ is observed.   In order to have the smallest MSE (mean square error), what is the predicted value of X?   (8%)
   (2) Let $Z = \dfrac{Y}{X}$.   (12%)
     (a) Find the joint pdf of (X, Z).
     (b) Find the marginal pdf of Z, $E(Z)$,   and $V(Z)$.

4. Consider two independent random samples:
   $X_i \sim N(\mu_1, \sigma_1^2), \ i = 1, \cdots, n_1$, and $Y_j \sim N(\mu_2, \sigma_2^2), \ j = 1, \cdots, n_2$.   Assume μ_1 and μ_2 are known.
   (1) Find the MLE of $\dfrac{\sigma_1^2}{\sigma_2^2}$.   (6%)
   (2) Find the exact $100(1-\alpha)$% confidence interval for $\dfrac{\sigma_1}{\sigma_2}$.   (8%)
   (3) Derive the size α LRT (likelihood ratio test) of $H_0 : \sigma_1^2 = \sigma_2^2$ against $H_a : \sigma_1^2 \neq \sigma_2^2$.   (16%)

備 註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。