考試科目	基礎數學	系所別	統計學系	考試時間	2 月 18 日（一）第一節

1. (20 pts) Let W be a subspace of R^n.

   (a) (5 pts) It's known that W is a subset of R^n and W itself is a vector space. What is a vector space?

   (b) (5 pts) Define the orthogonal complement of W, $W^{\perp}$.

   (c) (10 pts) Show that $R^n = W \oplus W^{\perp}$.

2. (10 pts) Let W be the subspace of the plane $3x - 2y + z = 0$ in R^3. Find the shortest distance of the vector $u = (2, 3, -1)$ to W.

3. (20 pts) Consider an experimental design. We have 3 factors of interest. Each treatment has two possible levels, H(HIGH) and L(LOW). Thus, there are 8 possible combinations in total. Define the following parameters for the response Y,

$$\begin{aligned} \mu &= \text{overall mean,} \\ \delta_1 &= \text{mean increment due to } X_1, \\ \delta_2 &= \text{mean increment due to } X_2, \\ \delta_3 &= \text{mean increment due to } X_3. \end{aligned}$$

The responses $Y_1, \cdots, Y_8$ under the 8 levels are expressed in the following way, where $\epsilon_i, i = 1, \cdots, 8$, are random errors.

	Treatment Level			Response model
Trial	X_1	X_2	X_3	Y
1	H	H	H	$Y_1 = \mu + \delta_1 + \delta_2 + \delta_3 + \epsilon_1$
2	H	H	L	$Y_2 = \mu + \delta_1 + \delta_2 - \delta_3 + \epsilon_2$
3	H	L	H	$Y_3 = \mu + \delta_1 - \delta_2 + \delta_3 + \epsilon_3$
4	H	L	L	$Y_4 = \mu + \delta_1 - \delta_2 - \delta_3 + \epsilon_4$
5	L	H	H	$Y_5 = \mu - \delta_1 + \delta_2 + \delta_3 + \epsilon_5$
6	L	H	L	$Y_6 = \mu - \delta_1 + \delta_2 - \delta_3 + \epsilon_6$
7	L	L	H	$Y_7 = \mu - \delta_1 - \delta_2 + \delta_3 + \epsilon_7$
8	L	L	L	$Y_8 = \mu - \delta_1 - \delta_2 - \delta_3 + \epsilon_8$

   (a) (10 pts) Express the model of the response variable in matrix form.

   (b) (10 pts) Find the least squared estimates of $\mu, \delta_1, \delta_2, \delta_3$.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

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For the following problems, show your calculation to receive full credit.

4. (10 pts) Evaluate the following integral:

$$\int_0^\infty \frac{\sin(x)}{x}\,dx.$$

5. (10 pts) Let

$$f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & (x,y) \neq (0,0), \\ 0 & (x,y) = (0,0). \end{cases}$$

Is f continuous at $x = y = 0$? (Give an explanation.)

6. (30 pts) Let $f(\mathbf{c}) = \mathbf{c}^T A \mathbf{c}$, where $\mathbf{c}$ is a 2×1 column vector and A is a 2×2 matrix:

$$\mathbf{c} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}.$$

(a) (20 pts) Find $\mathbf{c}^*$ that maximizes $f(\mathbf{c})$ subject to $c_1^2 + c_2^2 = 1$.

(b) (10 pts) Find $f(\mathbf{c}^*)$.

備 註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。