考試科目	統計學	系所別	經濟學系	考試時間	2月 18 日(一) 第 三 節

注意事項:
(1) 請依題號順序作答。
(2) 不可使用計算機。
(3) 答題若過程錯誤 (或沒有過程) 但答案正確, 將以「零分」計算。

1. (40%) The joint probability density function (pdf) of X and Y is given by

$$f_{X,Y}(x, y) = \begin{cases} c, & 0 \le x \le y \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

where c is an unknown constant.

(1) (5%) Find the value of c that makes this a valid pdf.

(2) (10%) Find the **marginal pdfs** for X and for Y.

(3) (5%) Are the random variables X and Y **independent**? Why or why not?

(4) (10%) Derive the **conditional mean** of Y, given $X = x$.

(5) (5%) Derive the "**conditional variance** of Y, given $X = x$.

(6) (5%) Find $\mathbb{P}(\frac{3}{4} \le Y \le \frac{7}{8} | X = \frac{1}{4})$. (You should clearly write down the reason.)

2. (25%) Let $X_1, X_2, \cdots, X_n$ be a random sample of size n from the distribution with probability density function (pdf):

$$f_X(x; \delta) = k \cdot \delta x^{-1+\delta}, \quad 0 < x < 1, \quad 0 < \delta < \infty,$$

with an unknown parameter δ. Then given a realization of data as $x_1, x_2, \cdots, x_n$,

(1) (5%) Find the value of k that makes this a valid pdf.

(2) (10%) Find the **method of moments estimator** and the corresponding **estimate** for δ.

(3) (10%) Find the **maximum likelihood estimator** the corresponding **estimate** for δ.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

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3. (15%) Let X_1 and X_2 be a random sample of size 2 from the uniform distribution on $\{2, 4, 6, 8\}$. Let $Y = X_1 + X_2$.

   (1) (10%) Find the **moment-generating function** of Y.

   (2) (5%) Find the **probability density function** (pdf) of Y.

4. (20%) X follows a normal distribution with mean μ_x and variance 36. Based on $n = 16$ observations and the corresponding sample mean $\bar{x}$, if we would like to test

$$H_o : \mu_x = 50 \quad \text{versus } H_1 : \mu_x > 50,$$

   with the **critical region:** $C = \{(x_1, x_2, \cdots, x_{16}) : \bar{x} \geq 53\}$.

   (1) (6%) What is the **size** of the test?

   (2) (8%) What is the $p-$**value** associated with $\bar{x} = 54.5$?

   (3) (6%) What is the probability of **Type II error** when $H_1 : \mu_x = 55$?

   (**Remark:** Be careful, the sample size here is less than 30.)

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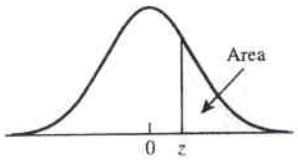
Table 4 Normal Curve Areas
Standard normal probability in right-hand tail
(for negative values of *z*, areas are found by symmetry)



	Second decimal place of *z*									
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
3.0	.00135									
3.5	.000 233									
4.0	.000 031 7									
4.5	.000 003 40									
5.0	.000 000 287									

From R. E. Walpole, *Introduction to Statistics* (New York: Macmillan, 1968).

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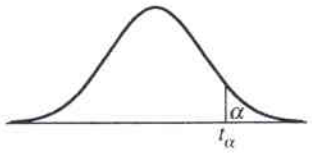
Table 5 Percentage Points of the *t* Distributions



$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	df
3.078	6.314	12.706	31.821	63.657	1
1.886	2.920	4.303	6.965	9.925	2
1.638	2.353	3.182	4.541	5.841	3
1.533	2.132	2.776	3.747	4.604	4
1.476	2.015	2.571	3.365	4.032	5
1.440	1.943	2.447	3.143	3.707	6
1.415	1.895	2.365	2.998	3.499	7
1.397	1.860	2.306	2.896	3.355	8
1.383	1.833	2.262	2.821	3.250	9
1.372	1.812	2.228	2.764	3.169	10
1.363	1.796	2.201	2.718	3.106	11
1.356	1.782	2.179	2.681	3.055	12
1.350	1.771	2.160	2.650	3.012	13
1.345	1.761	2.145	2.624	2.977	14
1.341	1.753	2.131	2.602	2.947	15
1.337	1.746	2.120	2.583	2.921	16
1.333	1.740	2.110	2.567	2.898	17
1.330	1.734	2.101	2.552	2.878	18
1.328	1.729	2.093	2.539	2.861	19
1.325	1.725	2.086	2.528	2.845	20
1.323	1.721	2.080	2.518	2.831	21
1.321	1.717	2.074	2.508	2.819	22
1.319	1.714	2.069	2.500	2.807	23
1.318	1.711	2.064	2.492	2.797	24
1.316	1.708	2.060	2.485	2.787	25
1.315	1.706	2.056	2.479	2.779	26
1.314	1.703	2.052	2.473	2.771	27
1.313	1.701	2.048	2.467	2.763	28
1.311	1.699	2.045	2.462	2.756	29
1.282	1.645	1.960	2.326	2.576	inf.

From "Table of Percentage Points of the *t*-Distribution." Computed by Maxine Merrington, *Biometrika*, Vol. 32 (1941), p. 300.

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