考試科目： 微分方程與線性代數

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※可使用計算器

1.答案以橫式由左至右書寫。2.請依題號順序作答。

1. Find the recurrence relation and use it to generate the first five terms of the series of the general solution for $y'' - xy' + y = 3$. (15%)

2. Solve the initial value problem (15%)

$$y'' + 4y = f(t);\quad y(0) = 1,\quad y'(0) = 0, \text{ with } f(t) = \begin{cases} 0, & \text{for } 0 \le t < 4 \\ 3, & \text{for } t \ge 4 \end{cases}.$$

3. Find the general solution of the equation (10%)

$$xy' + (x-2)y = 3x^3e^{-x}$$

4. Determine the relation between the nonzero constant parameters a and b such that the solution of the differential equation $y' = \dfrac{ax - y}{x + by}$ is

   (a) a straight line. (5%)

   (b) a circle. (5%)

5. Let $\mathbf{A} = \begin{bmatrix} -1 & 0 & 0 \\ 1 & 3 & -3 \\ 1 & 1 & -1 \end{bmatrix}$.

   (1) Determine a basis for the null space of **A**. (5%)

   (2) Please find all the eigenvalues of **A**. (5%)

   (3) Please find $e^{\mathbf{A}}$. (5%)

   (4) Please find a Hermitian **E** (i.e., $\mathbf{E} = \overline{\mathbf{E}}^T$) and a skew Hermitian matrix **F** (i.e., $\mathbf{F} = -\overline{\mathbf{F}}^T$) such that $\mathbf{A} = \mathbf{E} + \mathbf{F}$. (5%)

   (5) Find real numbers α, β, and γ such that $\mathbf{A}^3 = \alpha \cdot \mathbf{A}^2 + \beta \cdot \mathbf{A} + \gamma \cdot \mathbf{I}$. (6%)

6. Suppose that matrix $\mathbf{A} \in R^{n\times n}$ is nonsingular and $\lambda \in C$ is an eigenvalue of **A**. Prove that $1/\overline{\lambda}$ is an eigenvalue of $\mathbf{A}^{-1}$. (8%)

7. Let λ_1 and λ_2 be distinct real eigenvlaues of matrix $\mathbf{A} \in R^{n\times n}$. Suppose that $\mathbf{A}v_1 = \lambda_1 \mathbf{v}_1$ and $\mathbf{A}v_2 = \lambda_2 \mathbf{v}_2$ for some nonzero vectors $\mathbf{v}_1, \mathbf{v}_2 \in R^n$. Please show that $\mathbf{v}_1$ and $\mathbf{v}_2$ are linearly independent. (Note that $\lambda_1 \neq \lambda_2$) (8%)

8. Suppose that $\mathbf{A} \in C^{n\times n}$ is a unitary matrix. Prove that $|\det(\mathbf{A}^{-1})| = 1$. (8%)