



國立臺灣海洋大學一〇〇學年度研究所碩士班暨碩士在職專班入學考試試題

考試科目：工程數學

系所名稱：機械與機電工程學系碩士班微系統組 B 組(聯)、機械與機電工程學系碩士班熱流組(聯)

1. 答案以橫式由左至右書寫。2. 請依題號順序作答。

1. Find the general solution of the differential equation

$$x^2 y' = y^2 - xy + x^2,$$

hint: you may use either one of the following methods:

(a) by using $y = S(x) + z^{-1}$, where $S(x)$ is a particular solution.

(Riccati equation)

(b) by assuming $y = ux$. (homogeneous equation) (18%)

2. Solve the initial value problem:

$$y'' + 9y = 2 \sin(3t) ; y(0) = y'(0) = 0. \quad (16\%)$$

3. Find the recurrence relation and use it to generate the first five terms of the

Maclaurin series ($y(x) = \sum_{n=0}^{\infty} a_n x^n$) of the general solution.

$$y' + 2y = 1 \quad (16\%)$$

4. Given: $\phi = xy + z^2 + 7$; $F = xi + y^2j + z^4k$; $G = y^2i - xj + e^xj$

Fill out the following blanks:

$$(a) \|F\| = \underline{\hspace{2cm}} \quad (b) F \cdot G = \underline{\hspace{2cm}} \quad (c) \nabla \times G = \underline{\hspace{2cm}}$$

$$(d) \nabla \phi = \underline{\hspace{2cm}} \quad (e) \nabla^2 \phi = \underline{\hspace{2cm}} \quad (f) \nabla \cdot F = \underline{\hspace{2cm}}. \quad (18\%)$$

5. (a) Find the Fourier series of the function

$$f(x) = x \quad \text{for } -\pi \leq x \leq \pi$$

$$f(x) = f(x + 2\pi) \quad \text{for all } x.$$

(b) Determine what this series converges to. (16%)

6. Find the eigenvalues and the corresponding eigenfunctions of the following problem

$$x^2 y'' + xy' + \lambda y = 0 ; y(1) = y(e) = 0. \quad (16\%)$$