

考生作答須知：

一、本試卷共有兩部分，各佔 50 分。每題配分標示於題後，總分 100 分。

二、答案請於答案卷上依題號次序作答，題號務必標示清楚，並寫出計算過程，否則不予計分。

第一部分：微分 (50%)

1. Find the limits of the following: (10%)

$$(a) \lim_{n \rightarrow \infty} \frac{1^2 + 3^2 + \dots + (2n-1)^2}{n^3} \quad (b) \lim_{n \rightarrow \infty} \sqrt[n]{e+1}$$

2. Find the maximum of the following: (10%)

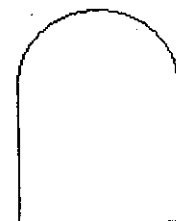
$$(a) y = \frac{x^2}{2^x} \quad (b) y = (x+2)^8 e^{-2(x+1)}$$

3. Find the sum of the following: (10%)

$$(a) S = \frac{1}{3} + \frac{1}{8} + \frac{1}{15} + \frac{1}{24} + \dots \quad (b) S = \frac{1}{2 \times 3 \times 4} + \frac{1}{4 \times 5 \times 6} + \frac{1}{6 \times 7 \times 8} + \dots$$

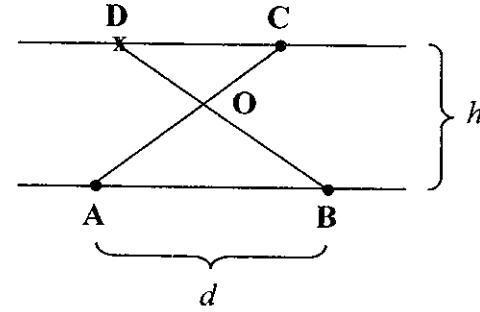
4. Consider a window having the following shape: (10%)

The window is a rectangle with a semi-circle on the top.
Assume the boundary of the window is fixed with length L .
What is the maximum possible area of the window?



5. Given the following figure of two parallel lines: (10%)

Points A , B , and C are fixed points on the lines, and distance between points A and B is d . Consider a point D on the left-hand side of point C . What is the maximum possible sum of the area of the two triangles $\triangle AOB$ and $\triangle COD$?



第二部分：積分 (50%)

6. What is "The Fundamental Theorem of Calculus"? (10%)

7. Evaluate each integral of the following: (20%)

(a) $\int x^2 e^x dx$. (b) $\int_{-\infty}^{\infty} x e^{-x^2} dx$. (c) $\int \frac{1}{x} dx$. (d) $\int \tan 2x dx$.

8. Determine the area under the curve $y = e^{-x^2}$ from 0 to any specified positive number t .

Use four series terms for the approximation. (10%)

9. When the region under the graph of $y = 1/x$ for $x \geq 1$ is revolved about the x axis, the solid created is called *Gabriel's horn*. Determine the volume of *Gabriel's horn*. (10%)