國立中山大學100學年度碩士班招生考試試題

科目：近代物理【物理系碩士班】

1. **(20%)** An electron is confined in the ground state in a one-dimensional box of width 10^{-10} m. Its energy is 38 eV.
   (a) **(10%)** Write the time-independent wave function corresponding to the state of the lowest possible energy.
   (b) **(10%)** Calculate the energy of the electron in its first excited state.
2. **(20%)** At the time t=0 the wave function for hydrogen atom is

$$\varphi(r,0)=\frac{1}{\sqrt{10}}(2\varphi_{100}+\sqrt{3}\varphi_{210}+\varphi_{211}+\sqrt{2}\varphi_{21-1})$$

   where the subscripts are values of the quantum numbers *n, l, m*. Ignore spin and radiative transitions.
   (a) **(8%)** Find the expectation value for the *z* component of the angular momentum, $\langle \vec{L}_z \rangle$, of this system.
   (b) **(8%)** Find the expectation value for the energy of this system.
   (c) **(4%)** What is the probability of finding the system with $l=+1$, $m=-1$ as a function of time?
3. **(15%)** The famed sodium doublet arises from the spin-orbit splitting of the sodium 3*p* level. The fine-structure splitting of the $2P_{3/2}$ and $2P_{1/2}$ levels in hydrogen is 2.13×10^{-3} eV. Estimate the magnetic field that the 2*p* electron in hydrogen experience. Assume $\vec{B}$ is parallel to z axis.
4. **(20%)** An electron in an atom has orbital angular momentum $\vec{L}_1$ with quantum number l_1=1, and a second electron has orbital angular momentum $\vec{L}_2$ with quantum number l_2=2.
   (a) **(10%)** What are the possible quantum number for the total orbital angular momentum $\vec{L}=\vec{L}_1+\vec{L}_2$.
   (b) **(10%)** What are the possible quantum number *j* for the combination $\vec{J}=\vec{L}+\vec{S}$.
5. **(25%)** Investigate the Zeeman spectrum produced by hydrogen atoms initially in the *n*=2 state. Assume the atoms to be in a magnetic field of magnitude $\vec{B}$=2.00 T, and choose the *z*-axis along the direction of $\vec{B}$.
   (a) **(6%)** Express and calculate the total magnetic energy of an electron with orbital and spin contributions. $[\frac{e\hbar}{2m_e}=9.27\times10^{-24}(J/T)=5.79\times10^{-5}(eV/T)]$
   (b) **(7%)** Draw the energy levels for *n*=1 and *n*=2 without spin consideration. Write each energy state by (n, l, m_l) and each energy level in the unit of eV.
   (c) **(7%)** Draw the energy levels for *n*=1 and *n*=2 with spin consideration. Write each energy state by (n, l, m_l, m_s) and each energy level in the unit of eV.
   (d) **(5%)** Indicate the possible transition in (b) and (c) for an electron excited to the *n*=2 state of hydrogen. Selection rules should be taken into account.