

(每題 10 分, 共 10 題)

1. Evaluate the integral $\int \delta[(x-a)(x-b)] f(x) dx$, where $\delta[\dots]$ is the Dirac delta function and the range of integration includes the points a , and b ($a \neq b$). (10%)
2. If a vector \vec{F} is given by $\vec{F} = (x^2 + y^2 + z^2)^n (x\hat{e}_x + y\hat{e}_y + z\hat{e}_z)$, where \hat{e}_i is the unit vector along the i direction, find (a) $\nabla \cdot \vec{F}$. (5%) (b) $\nabla \times \vec{F}$. (5%)
3. Resolve the circular cylindrical unit vectors \hat{e}_ρ , \hat{e}_ϕ , and \hat{e}_z into their Cartesian components \hat{e}_x , \hat{e}_y , and \hat{e}_z . (10%)
4. Evaluate the function of matrix $\exp\{i\sigma_2\theta\}$, where $\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ in which i is the imaginary unit. (10%)
5. For a resistance-inductance circuit Kirchoff's law leads to the differential equation $L \frac{dI(t)}{dt} + RI(t) = V$ for the current $I(t)$, where L is the inductance, R is the resistance, and V is the time-independent input voltage, all constant. If the initial condition is $I(0) = 0$, find the solution of the current $I(t)$. (10%)

6. Find the Fourier series expansion of the function $f(x) = x$ in the specific interval $-\pi \leq x \leq \pi$. (10%)

7. Evaluate the integral $\int_0^{2\pi} \frac{d\theta}{1 - 2\alpha \cos \theta + \alpha^2}$, $\alpha \in (0, 1)$. (10%)

8. Find the Fourier transformation of the function $f(t) = \sin \omega_0 t$, where ω_0 is a constant. (10%)

9. Find the Laurent series of the function of a complex variable

$$f(z) = \frac{1}{z^2(z-i)} \text{ in the intervals}$$

(a) $0 < |z-i| < 1$, (5%) (b) $|z-i| > 1$. (5%).

10. Find the general solution of the partial differential

$$\text{equation} \begin{cases} \frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}, & x \in (0, l), t > 0, \\ u_x(0, t) = u_x(l, t) = 0, & t > 0, \\ u(x, 0) = \varphi(x), & x \in (0, l), \end{cases}$$

where $u_x = \frac{\partial u}{\partial x}$. (10%)