

國立交通大學 100 學年度碩士班考試入學試題

科目：微積分(5011)

考試日期：100 年 2 月 17 日 第 2 節

系所班別：管理科學系 組別：管科系

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【可使用計算機】*作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符!!

1. (10 %) Find all k such that the limit

$$\lim_{x \rightarrow \infty} \frac{\int_0^x (e^{2t} \sqrt{t^6 + 1}) dt}{x^k e^{2x}}$$

has a nonzero limit. Then find its value.

2. (10 %) Evaluate the integral $\int_0^9 \int_{\sqrt{y}}^3 \sin(\pi x^3) dx dy$.
3. (a) (5 %) Find the length of the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$.
 (b) (5 %) Find the area of the surface generated by revolving the curve of (a) about the x -axis.
4. (10 %) Find $\frac{d^{100}}{dx^{100}} \Big|_{x=0} (\sin^{-1} x)$ and $\frac{d^{2011}}{dx^{2011}} \Big|_{x=0} (\sin^{-1} x)$.
5. (10 %) Let $f(x) = 3x^5 - 5x^3 + 3$. Find the intervals on which f is increasing, decreasing, convex up, convex down. At what points, if any, does the graph of f have a local maximum, local minimum, or inflection point? Sketch the graph of f .
6. (10 %) For positive real number c , define the sequence a_n recursively by fixing $a_1 > 0$, and define $a_{n+1} = \sqrt{c + a_n}$. Prove or disprove that the sequence a_n converges to the solution of the quadratic equation $x^2 - x - c = 0$.
7. (10 %) Prove that $1 + \frac{x^2}{2} - \frac{x^4}{8} \leq \sqrt{1 + x^2} \leq 1 + \frac{x^2}{2}$, for all x .
8. Let $f(x) = 3 + x + e^x$, $x \in \mathbb{R}$.
 (a) (2 %) Show that f is invertible.
 (b) (8 %) Find $(f^{-1})'(4)$ and $(f^{-1})''(4)$.
9. (10 %) The plane $x + y + 2z = 2$ intersects the paraboloid $z = x^2 + y^2$ in an ellipse. Find the points on this ellipse that are nearest to the origin.
10. (10 %) Find the area of the region that consists of all points that lie within the circle $r = 2 \cos \theta$ but outside the circle $r = 1$.