

1. (20%) Find the general solution to the differential equation

$$\frac{dy}{dx} = \frac{4 \sin x - y}{\tan x + 3y^2 \sec x}$$

You may express the solutions in implicit form.

- 2.(20%) Find the general solution to the system

$$\dot{x} = \frac{7}{2}x - \frac{1}{2}y + t,$$

$$\dot{y} = \frac{1}{2}x + \frac{5}{2}y + t.$$

- 3.(20%) Consider the system

$$\dot{x} = x^2 + 4y^2 - 16,$$

$$\dot{y} = 2xy.$$

Find the equalibria and determine their stability types. Sketch the phase diagram.

4. (20%) Let $x(t), y(t)$ be a solution to

$$\dot{x} = x^3 - xy^2,$$

$$\dot{y} = y^3 + 3x^2y.$$

Suppose $x(t), y(t)$ are nontrivial real functions. Show that $|x(t)|^2 + |y(t)|^2$ tends to infinity in finite time. Determine the blow up time in terms of initial data.

- 5.(20%) Let $a, b,$ and c be positive constants. Show that the any two different solutions of

$$ay'' + by' + cy = g(t)$$

approach each other as time approaches infinity.

試題隨卷繳回