

開南大學 100 學年度碩士班招生考試試題卷

科目：微積分（碩士班）

用紙第 1 頁共 4 頁

請將正確答案填寫於答案卷中

1. (3%) Graph the function with a graphing utility and use it to predict the limit. Check your work either by using the table feature of the graphing utility or by finding the limit algebraically.

$$\lim_{x \rightarrow 9} \frac{x^3 - 9x^2}{2x^2 - 20x + 18}$$

- a) 0 b) 81 c) $\frac{81}{16}$ d) $\frac{81}{7}$ e) does not exist

2. (3%) Determine whether the given function is continuous. If it is not, identify where it is discontinuous.

$$y = 4x^2 - 3x + 7$$

- a) discontinuous at $x = 3$ b) discontinuous at $x = 0$
c) discontinuous at $x = -3$ d) discontinuous at $x = 6$
e) continuous everywhere

3. (3%) Determine the following limit.

$$\lim_{x \rightarrow 1} \frac{1}{x - 1}$$

- a) 0 b) does not exist c) 1 d) -1 e) -2

4. (3%) Find the derivative of the following function using the limiting process.

$$f(x) = \sqrt{3x+6}$$

- a) $f'(x) = \frac{3}{2\sqrt{3x+6}}$ b) $f'(x) = -\frac{3}{2\sqrt{3x+6}}$ c) $f'(x) = \frac{3}{2}(3x+6)^{1/2}$
d) $f'(x) = -\frac{3}{\sqrt{3x+6}}$ e) either B or D

5. (3%) Write the equation of the line tangent to the graph of $f(x)$ at $x = ?$.

$$f(x) = 2x^2 + 6x$$

- a) $y = 2x - 3$ b) $y = 2x + 3$ c) $y = 2x$ d) $y = 2x - 2$ e) $y = 2x + 2$

6. (3%) Find $f'(x)$ of the function $f(x) = \frac{7x^7 - 4x^6 + 3}{x^6}$.

- a) $\frac{x^7 - 18}{x^7}$ b) $\frac{7x^6 - 3}{x^6}$ c) $\frac{x^7 + 3}{x^7}$ d) $\frac{7x^6 + 18}{x^6}$ e) $\frac{7x^7 - 18}{x^7}$

請翻背面繼續作答

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7. (3%) The variable cost for manufacturing an electrical component is \$9.75 per unit, and the fixed cost is \$600. Write the cost C as a function of x , the number of units produced. Find the derivative of the cost function.

- a) 9.75 b) 600 c) 609.75 d) 590.25 e) 0

8. (5%) The graph shows the number of visitors V to a national park in hundreds of thousands during a one-year period, where $t = 1$ represents January. Estimate the rate of change of V over the interval $[4, 7]$. Round your answer to the nearest hundred thousand visitors per year.



- a) 177.27 hundred thousand visitors per year b) 390.00 hundred thousand visitors per year c) 250.00 hundred thousand visitors per year
 d) 325.00 hundred thousand visitors per year e) 650.00 hundred thousand visitors per year

9. (3%) Evaluate the integral $\int 20x^7 dx$.

- a) $140x^6 + C$ b) $160x^8 + C$ c) $\frac{10}{3}x^6 + C$ d) $\frac{5}{2}x^8 + C$ e) $\frac{20}{7}x^7 + C$

10. (3%) Use algebra to rewrite the integrand; then integrate and simplify.

$$\int \frac{x + 2}{\sqrt{x}} dx$$

- a) $\frac{2}{3}x\sqrt{x} + 2\sqrt{x} + C$ b) $\frac{2}{3}x\sqrt{x} + 4\sqrt{x} + C$ c) $\frac{2}{3}x\sqrt{x} + 6\sqrt{x} + C$

- d) $\frac{1}{2}x^2 + 6\sqrt{x} + C$ e) $\frac{1}{2}x^2 + 4\sqrt{x} + C$

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11. (3%) Evaluate the integral $\int 7x^8 \sqrt{x^9 + 7} dx$.

- a) $\frac{14}{27}(x^9 + 7)^{3/2} + C$ b) $\frac{14}{9}(x^9 + 7)^{3/2} + C$ c) $\frac{7}{6}(x^9 + 7)^{3/2} + C$
d) $\frac{7}{9}x^9 + C$ e) $\frac{7}{8}x^9 + C$

12. (3%) Use the values $\int_0^4 f(x)dx = 8$ and $\int_0^4 g(x)dx = 1$ to evaluate the definite

$$\text{integral } \int_0^4 [f(x) - 2g(x)] dx .$$

- a) 10 b) 6 c) 7 d) 9 e) 8

13. (4%) After test-marketing a new menu item, a fast-food restaurant predicts that sales of the new item

will grow according to the model $\frac{dS}{dt} = \frac{4t}{(t+6)^2}$, where t is the time in weeks and S is the sales (in

thousands of dollars). Find the sales of the menu item at 30 weeks.

- a) 4.015 thousand dollars b) 4.103 thousand dollars c) 4.188 thousand dollars
d) 3.834 thousand dollars e) 3.925 thousand dollars

14. (3%) Find the area of the region bounded by the graphs of the given equations. Round your answer to two decimal places.

$$y = \frac{8x}{\sqrt{4x+2}}, \quad y = 0, \quad x = 8$$

- a) 28.15 square units b) 56.31 square units c) 18.77 square units
d) 84.46 square units e) 37.54 square units

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15. (5%) Evaluate the definite integral

$$\int_1^3 x^3 \ln x dx$$

Round your answer to three decimal places.

a) 17.747 b) 18.810 c) 16.144 d) 17.247 e) 16.484

16. 判斷下列無窮級數是否收斂

(a) $\sum_{n=1}^{\infty} \frac{1}{n}$ (b) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$ (c) $\sum_{n=1}^{\infty} \frac{1}{n^2}$ (d) $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ (16%)

17. 求 $\sum_{n=2}^{\infty} \frac{1}{n(n-1)}$ 。 (10%)

18. 假設 $f(x) = x e^{-x}$ 為定義在區間 $(-\infty, \infty)$ 上，求 f 在 $a=0$ 的三階泰勒多項式。 (12%)

19. 求函數 $f(x, y) = x^2 + 2y^2$ 在限制條件 $x - y - 1 = 0$ 之下的極大值或極小值。 (12%)

請翻背面繼續作答