國立台灣科技大學一百學年度碩士班招生試題

系所組別： 機械工程系碩士班甲組、乙組、丙組、丁組

科　　目： 工程數學

（總分為100分）

1. Find the general solution of the differential equation

$$y' = (-2x + y)^2 - 7$$

(20%)

2. Solve the initial value problem

$$y'' - 4y' + 4y = f(t); y(0) = y'(0) = 0, with$$

$$f(t) = \begin{cases} t & for\ 0 \le t < 3 \\ t+2 & for\ t \ge 3 \end{cases}$$

(20%)

3. Use an *orthogonal transformation* to transform the real quadratic form

$Q = 4x_1x_2 + 4x_2x_3 + 4x_3x_1$ into its canonical form.

Write down the canonical form and the transformation matrix. (20%)

4. Verify Green's theorem for the case where the vector field

$$\vec{F}(x, y) = (xy + y^2)\vec{i} + x^2\vec{j},$$

and the region is bounded by the curves $y = x$ and $y = x^2$. (20%)

(Note: Calculate each side of Green's theorem, respectively, and show that both results are identical.)

5. Starting from the separation of variables, solve the boundary-value problem

$$\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2} = 0, \quad 1 \le r \le 2, \quad 0 \le \theta \le \pi$$

with $w(r,0) = w(r,\pi) = w(1,\theta) = 0$, and $w(2,\theta) = \sin^3\theta$. (20%)