國立臺北科技大學100學年度碩士班招生考試

系所組別：2220 電腦與通訊研究所乙組

第一節　工程數學　試題

注意事項：

1. 本試題共六題，配分共100分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

一、

True and False: There are two $N \times N$ matrices **A** and **B**. Show that if the following statements are true or false. Note that det(·) represents the determinant, $(^T)$ represents the transpose, and tr(·) represents the trace.

1. $\det(-\mathbf{A}) = -\det(\mathbf{A})$ (3%)
2. $(\mathbf{AB})^2 = \mathbf{A}^2\mathbf{B}^2$ (3%)
3. $(\mathbf{A}+\mathbf{B})^2 = \mathbf{A}^2 + 2\,\mathbf{AB} + \mathbf{B}^2$ (3%)
4. $(\mathbf{A}+\mathbf{B})^T = \mathbf{A}^T + \mathbf{B}^T$ (3%)
5. $\mathrm{tr}(\mathbf{A}+\mathbf{B}) = \mathrm{tr}(\mathbf{A}) + \mathrm{tr}(\mathbf{B})$ (3%)
6. If both **A** and **B** are invertible, then $(\mathbf{A}+\mathbf{B})$ is also invertible. (3%)
7. If both **A** and **B** are invertible, then **AB** is also invertible. (3%)
8. If **AB** is invertible, then both **A** and **B** are invertible. (3%)
9. If λ is an eigenvalue of **A**, then λ^{-1} is an eigenvalue of $\mathbf{A}^{-1}$. (3%)
10. If λ is an eigenvalue of **A**, then λ is also an eigenvalue of $\mathbf{A}^T$. (3%)

二、

There is a quantity $Q = \mathbf{b}^{*T}\mathbf{a}$, where **a** and **b** are $N\times 1$ vectors, and $(^*)$ represents the complex conjugate. Find $\frac{\partial Q}{\partial \mathbf{a}}$ and $\frac{\partial Q}{\partial \mathbf{a}^*}$. (10%)

三、

Find $\mathbf{A}^{-0.5}$, where **A** is a matrix whose eigenvalues are 8 and 18, and the corresponding eigenvectors are $\begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$ and $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$, respectively. (10%)

四、

Assume that there are two events, A_1 and A_2 in a probability space S, satisfying $A_1 \cup A_2 = S$, and $A_1 \cap A_2 = \{\phi\}$. The probabilities of occurring A_1 and A_2 are equal. If another event $B \subset S$, satisfies $P(A_1 | B) = 0.8$ and $P(B | A_2) = 0.1$, then

1. $P(B | A_1) = ?$ (5%)
2. $P(B) = ?$ (5%)
3. Are A_2 and B mutually exclusive? (5%)
4. Are A_2 and B independent? (5%)

五、

The random variables X and Y are jointly Normal with density function:

$f_{X,Y}(x,y) = \frac{1}{8\pi}\exp\left\{-\frac{1}{8}\left[(x-6)^2 + (y-9)^2\right]\right\}$. Find $E\{E\{Y^2 | X\}\}$ and $\mathrm{Var}(3X + 4Y)$, where $E\{\cdot\}$ and $\mathrm{Var}(\cdot)$ represent the expectation and variance, respectively. (10%)

六、

There are three random variables X, Y, and Z, where $Z = X + Y$. Find the probability density (or mass) function of Z for each of the following cases.

1. Suppose that X and Y are independent Poisson variables, (10%)

   $P(X = x) = e^{-3}\frac{3^x}{x!}$, where $x = 0, 1, 2, \ldots, \infty$,

   $P(Y = y) = e^{-2}\frac{2^y}{y!}$, where $y = 0, 1, 2, \ldots, \infty$.

2. Suppose that X and Y are independent continuous variables uniformly distributed between 0 and 1. (10%)