# 國立臺北科技大學 100 學年度碩士班招生考試

## 系所組別：2111、2112、2120、2130 電機工程系碩士班

## 甲、乙、丙組 第二節　工程數學　試題



**注意事項**：

1. 本試題共 7 題，配分共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

1. (15%) Find $f(x)$, let $y=f(x)$ be the solution of

$$(\frac{dy}{dx})^2 + y^2\frac{dy}{dx} - y\frac{d^2y}{dx^2} = 0$$

2. (15%) Find inverse Laplace transform of $F(s) = \frac{b}{as^2}\tanh\left(\frac{as}{2}\right)$;

hint: $f(t)$ is a periodic function, period=$2a$

3. (20%) Solve $x^3(\sin x)\,y''' - (3x^2 \sin x + x^3 \cos x)\,y'' + (6x \sin x + 2x^2 \cos x)\,y' - (6 \sin x + 2x \cos x)\,y = 0$, given one particular solution $y=x$

4. (10%) Let $A = \begin{bmatrix} 1 & 1 & 2 & 0 \\ 2 & 1 & 3 & 1 \\ 1 & 2 & 0 & 1 \end{bmatrix}$. Find the nullity and the null space of $A$.

5.(10%) Let $L$ be the linear mapping in $R^3$ defined by $L(x) = Ax$ corresponding to the standard basis, where $A = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -2 \\ 2 & -1 & -1 \end{bmatrix}$, and let $v_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, and $v_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ form another basis $[v_1, v_2, v_3]$. Find the matrix $B$ representing $L$ with respect to $[v_1, v_2, v_3]$.

6. Let $A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 2 & 0 & 1 & 1 \\ 2 & 7 & 7 & 1 \\ 2 & 1 & 7 & 1 \end{bmatrix}$

(a)(10%) Find the determinant and all eigenvalues of $A$;

(b)(10%) Find the inverse of $A$.

7. (10%) Prove in English that "The system of $n$ linear equations in $n$ unknowns $Ax = b$ has a unique solution of and only if $A$ is nonsingular.".

(Note : No credit will be given if the answer is given in Chinese)