國立臺北科技大學100學年度碩士班招生考試

系所組別：1310、1320、1330 車輛工程系碩士班甲、乙、丙組

第二節　工程數學　試題

注意事項：

1. 本試題共十題，配分共100分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

一、(10%) Find the general solution of the differential equation

$y' = (x+y+1)^2$. (hint: $\int \frac{1}{x^2+1}dx = \tan^{-1} x$)

二、(10%) Prove that a particular solution of a differential equation

$y'' + p(x)y' + q(x)y = r(x)$ can be found by $y_p = -y_1 \int \frac{y_2 r}{W} dx + y_2 \int \frac{y_1 r}{W} dx$,

where y_1, y_2 form a basis of solutions of the homogeneous equation

$y'' + p(x)y' + q(x)y = 0$ and $W = y_1 y_2' - y_2 y_1'$.

三、(10%) Find the general solution of the differential equation

$x^2 y'' - 2xy' + 2y = x^3 \sin x$. (hint: $\int x \sin x dx = \sin x - x\cos x$)

四、(10%) Solve he differential equation $y' + 3y = 2\delta(t)$ with the initial condition $y(t) = 1$, when $t < 0$, where $\delta(t)$ is the unit impulse function defined as

$$\delta(t) = \lim_{k\to 0} f_k(t), \text{ and } f_k(t) = \begin{cases} 1/k, & \text{if } 0 \le t \le k \\ 0, & \text{otherwise} \end{cases}.$$

五、Find the Laplace transform of the following functions.

1. (5%) $te^t \sin(t)$

(hint: $L[tf(t)] = -\frac{dF(s)}{ds}, L[e^{at} f(t)] = F(s-a)$, where $F(s) = L[f(t)]$)

2. (5%) $u(t-1)\sin(t)$, where $u(t-a)$ is the unit step function defined as

$u(t-a) = \begin{cases} 0, & \text{if } t < a \\ 1, & \text{if } t \ge a \end{cases}$. (hint: $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$)

六、(10%) Find the eigenvalues and the eigenvectors of $\mathbf{A} = \begin{bmatrix} 3 & -1 \\ 1 & 1 \end{bmatrix}$.

七、(10%) Find the general solution of the system of differential equations

$$\begin{aligned} y_1' &= 3y_1 - y_2 \\ y_2' &= y_1 + y_2 \end{aligned}.$$

八、(10%) Evaluate $e^{\mathbf{A}}$, where $\mathbf{A} = \begin{bmatrix} 3 & -1 \\ 1 & 1 \end{bmatrix}$ and $e^{\mathbf{A}} = \mathbf{I} + \mathbf{A} + \frac{1}{2!}\mathbf{A}^2 + \cdots$.

Each entry of the resulted matrix can be expressed by the exponential function.

九、(10%) Find the equation of the intersection line of two planes

$x + y + z = 6$ and $x - y + z = 2$.

十、(10%) The inverse of a nonsingular $n \times n$ matrix $\mathbf{A} = [a_{ij}]$ can be evaluated by $\mathbf{A}^{-1} = \frac{1}{|\mathbf{A}|}[C_{ij}]^T$, where C_{ij} is the cofactor of a_{ij}, $[C_{ij}]^T$ the transpose of $[C_{ij}]$ and $|\mathbf{A}|$ the determinant of $\mathbf{A}$. Evaluate $|\mathbf{A}|$ and $\mathbf{A}^{-1}$

when $[C_{ij}] = \begin{bmatrix} 8 & 0 & 0 & 0 \\ 9 & 4 & 0 & 0 \\ 0 & 5 & 2 & 0 \\ 7 & 6 & 3 & 1 \end{bmatrix}$.