考試科目	微積分	系所別	國貿系國際經濟·國際財管·國際企管與行銷組	考試時間	2月2日(星期五) 第三節

Multiple choice questions (4 points each). 選擇題請在答案卡上作答，否則不予計分。

1. Let $f(x) = x^3 - 3x + 2$. Which of the following statements is true?
   (a) $f'(0) \leq 1$ and $f''(0) \leq 1$.
   (b) $f'(0) \leq 1$ and $f''(0) > 1$.
   (c) $f'(0) > 1$ and $f''(0) \leq 1$.
   (d) $f'(0) > 1$ and $f''(0) > 1$.
   (e) $f'(0)$ does not exist.

2. Let $f(x) = e^x \cos(x)$. Which of the following statements is true?
   (a) $f'(0) \leq 1$ and $f''(0) \leq 1$.
   (b) $f'(0) \leq 1$ and $f''(0) > 1$.
   (c) $f'(0) > 1$ and $f''(0) \leq 1$.
   (d) $f'(0) > 1$ and $f''(0) > 1$.
   (e) $f'(0)$ does not exist.

3. Suppose that f is a differentiable function on $(-\infty, \infty)$ such that $f(1) = 1$ and $f'(1) = 2$. Let $h(x) = x/f(x^2)$ for $x \in (-\infty, \infty)$. Which of the following statements is true?
   (a) $h'(1) \leq 1$.
   (b) $1 < h'(1) \leq 2$.
   (c) $2 < h'(1) \leq 3$.
   (d) $h'(1) > 3$.
   (e) $h'(1)$ may or may not exist.

4. Suppose that f is a differentiable function on $(-\infty, \infty)$ such that $f(1) = 1$ and $f'(1) = 2$. Let $h(x) = f(f(x)) + \ln(x)$ for $x > 0$. Which of the following statements is true?
   (a) $h'(1) \leq 1$.
   (b) $1 < h'(1) \leq 2$.
   (c) $2 < h'(1) \leq 3$.
   (d) $h'(1) > 3$.
   (e) $h'(1)$ may or may not exist.

5. Let $f(x) = x + \sin(x)$ and $g(x) = x + \cos(x)$. Which of the following statements is true?
   (a) $-\infty < \lim_{x\to\infty} g(x)/f(x) \leq 1$.
   (b) $1 < \lim_{x\to\infty} g(x)/f(x) \leq 2$.
   (c) $2 < \lim_{x\to\infty} g(x)/f(x) \leq 3$.
   (d) $3 < \lim_{x\to\infty} g(x)/f(x) < \infty$.
   (e) $\lim_{x\to\infty} g(x)/f(x)$ does not exist.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

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6. Let $f(x) = \sin(x-3)$ and $g(x) = \ln(x-3)$. Which of the following statements is true?
   (a) $\lim_{x\to 3+} f(x)g(x)$ does not exist.
   (b) $-\infty < \lim_{x\to 3+} f(x)g(x) \le -3$.
   (c) $-3 < \lim_{x\to 3+} f(x)g(x) \le -1$.
   (d) $-1 < \lim_{x\to 3+} f(x)g(x) \le 1$.
   (e) $1 < \lim_{x\to 3+} f(x)g(x) < \infty$.

7. Let $f(x) = x\ln(1+x)$ and $g(x) = x\ln(x)$ for $x > 0$. Which of the following statements is true?
   (a) $\lim_{x\to\infty}(f(x) - g(x))$ does not exist.
   (b) $3 < \lim_{x\to\infty}(f(x) - g(x)) < \infty$.
   (c) $0 < \lim_{x\to\infty}(f(x) - g(x)) \le 3$.
   (d) $-3 < \lim_{x\to\infty}(f(x) - g(x)) \le 0$.
   (e) $-\infty < \lim_{x\to\infty}(f(x) - g(x)) \le -3$.

8. Suppose that $f'(x) = xe^x$ for $x \in (-\infty, \infty)$. Which of the following statements is true?
   (a) f is strictly increasing on the interval $(-1, \infty)$.
   (b) f is strictly decreasing on the interval $(0, \infty)$.
   (c) f has a minimum on the interval $(-1, \infty)$.
   (d) f has a maximum on the interval $(0, \infty)$.
   (e) None of the above statements holds true.

9. Let $f(x) = x^2 - \int_0^x t\cos(t)dt$ for $x \in (-\infty, \infty)$. Which of the following statements is true?
   (a) $f'(\pi/2) \le -2$.
   (b) $-2 < f'(\pi/2) \le -1$.
   (c) $-1 < f'(\pi/2) \le 0$.
   (d) $f'(\pi/2) > 0$.
   (e) None of the above statements holds true.

10. Let $f(x) = x^2 - \int_0^x t\cos(t)dt$ for $x \in (-\infty, \infty)$. Which of the following statements is true?
   (a) $\lim_{x\to 0} f(x)/x$ does not exist.
   (b) $-\infty < \lim_{x\to 0} f(x)/x \le 1$.
   (c) $1 < \lim_{x\to 0} f(x)/x \le 2$.
   (d) $2 < \lim_{x\to 0} f(x)/x \le 3$.
   (e) $3 < \lim_{x\to 0} f(x)/x < \infty$.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

考試科目	微積分	系所別	國貿系國際經濟·國際財管·國際企管與行銷組	考試時間	2月2日(星期五) 第三節

11. Which of the following statements is true?
    (a) $\int_0^1 x^{-2}dx \le 0.5$ and $\int_0^\infty e^{-x}dx \le 1$.
    (b) $\int_0^1 x^{-2}dx \le 0.5$ and $\int_0^\infty e^{-x}dx > 1$.
    (c) $0.5 < \int_0^1 x^{-2}dx \le 2$ and $\int_0^\infty e^{-x}dx \le 1$.
    (d) $0.5 < \int_0^1 x^{-2}dx \le 2$ and $\int_0^\infty e^{-x}dx > 1$.
    (e) None of the above statements holds true.

12. Suppose that $\{a_n\}_{n=1}^\infty$ is a sequence such that $a_1 = 1$ and $a_{n+1} = 0.5a_n + n^2$ for $n \ge 1$. Which of the following statements is true?
    (a) $-\infty < \lim_{n\to\infty} a_n \le 0$.
    (b) $0 < \lim_{n\to\infty} a_n \le 1$.
    (c) $1 < \lim_{n\to\infty} a_n \le 2$.
    (d) $2 < \lim_{n\to\infty} a_n < \infty$.
    (e) None of the above statements holds true.

13. Which of the following statements is true?
    (a) $0 \le \sum_{n=1}^\infty 3^{-n} \le 1$.
    (b) $1 < \sum_{n=1}^\infty 3^{-n} \le 2$.
    (c) $2 < \sum_{n=1}^\infty 3^{-n} \le 3$.
    (d) $3 < \sum_{n=1}^\infty 3^{-n} < \infty$.
    (e) None of the above statements holds true.

14. Which of the following statements is true?
    (a) $\sum_{n=1}^\infty (n+1)/(2n^3+n+1) < \infty$ and $\sum_{n=1}^\infty (-1)^n \cdot n$ diverges.
    (b) $\sum_{n=1}^\infty (n+1)/(2n^3+n+1) < \infty$ and $\sum_{n=1}^\infty (-1)^n \cdot n$ converges conditionally.
    (c) $\sum_{n=1}^\infty (n+1)/(2n^3+n+1) = \infty$ and $\sum_{n=1}^\infty (-1)^n \cdot n$ diverges.
    (d) $\sum_{n=1}^\infty (n+1)/(2n^3+n+1) = \infty$ and $\sum_{n=1}^\infty (-1)^n \cdot n$ converges conditionally.
    (e) $\sum_{n=1}^\infty (-1)^n \cdot n$ converges absolutely.

15. Let $a_n = (n!)^2$ and $b_n = n^2$. Which of the following statements is true?
    (a) $\sum_{n=1}^\infty b_n/a_n < \infty$ and $\sum_{n=1}^\infty (-1)^n/n$ diverges.
    (b) $\sum_{n=1}^\infty b_n/a_n < \infty$ and $\sum_{n=1}^\infty (-1)^n/n$ converges conditionally.
    (c) $\sum_{n=1}^\infty b_n/a_n = \infty$ and $\sum_{n=1}^\infty (-1)^n/n$ diverges.
    (d) $\sum_{n=1}^\infty b_n/a_n = \infty$ and $\sum_{n=1}^\infty (-1)^n/n$ converges conditionally.
    (e) $\sum_{n=1}^\infty (-1)^n/n$ converges absolutely.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

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16. Let $f_n(x) = \sum_{k=1}^{n} kx^{k-1}$ for $x \in (-\infty, \infty)$. Which of the following statements is true?
    (a) $\lim_{n\to\infty} f_n(x)$ exists for every $x \in (-\infty, \infty)$.
    (b) $\lim_{n\to\infty} f_n(x)$ does not exist if $|x| > 0.5$.
    (c) $\lim_{n\to\infty} f_n(x)$ does not exist if $|x| > 1$.
    (d) $\lim_{n\to\infty} f_n(x)$ does not exist if $x \neq 0$.
    (e) None of the above statements holds true.

17. Let $f_n(x) = 1 + \sum_{k=1}^{n} x^k/k$ for $n \geq 1$. Let $S = \{x : \lim_{n\to\infty} f_n(x) \text{ exists}\}$ and $f(x) = \lim_{n\to\infty} f_n(x)$ for $x \in S$. Which of the following statements is true?
    (a) $0.5 \notin S$.
    (b) $0.5 \in S$ and $f'(0.5)$ does not exist.
    (c) $0.5 \in S$ and $f'(0.5) < 2$.
    (d) $0.5 \in S$ and $f'(0.5) > 2$.
    (e) $0.5 \in S$ and $f'(0.5) = 2$.

18. Let $f(x, y) = \sin(xy) + 2x + y$ for $x, y \in (-\infty, \infty)$. Which of the following statements is true?
    (a) $f_x(0, 2) > 1$ and $f_{xx}(0, 2) > 5$.
    (b) $f_x(0, 2) > 1$ and $f_{xx}(0, 2) \leq 5$.
    (c) $f_x(0, 2) \leq 1$ and $f_{xx}(0, 2) > 5$.
    (d) $f_x(0, 2) \leq 1$ and $f_{xx}(0, 2) \leq 5$.
    (e) None of the above statements holds true.

19. Let $f(x, y) = \int_y^x \frac{y}{2y + t} dt$ for $0 < y < x$. Which of the following statements is true?
    (a) $f_x(2, 1) > 0.5$ and $f(x, y) > 0$ for $0 < y < x$.
    (b) $f_x(2, 1) > 0.5$ and $f(x, y) < 0$ for $0 < y < x$.
    (c) $f_x(2, 1) < 0$ and $f(x, y) > 0$ for $0 < y < x$.
    (d) $f_x(2, 1) < 0$ and $f(x, y) < 0$ for $0 < y < x$.
    (e) $0 \leq f_x(2, 1) \leq 0.5$.

20. Let $f(x, y) = xy + x + y$ for $x, y \in (-\infty, \infty)$, $D_1 = \{(x, y) : 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1\}$ and $D_2 = \{(x, y) : x > 0, y > 0 \text{ and } x^2 + y^2 \leq 1\}$. Which of the following statements is true?
    (a) $\int_{D_1} f(x, y)d(x, y) \leq 1$ and $\int_{D_2} f(x, y)d(x, y) > \int_{D_1} f(x, y)d(x, y)$.
    (b) $\int_{D_1} f(x, y)d(x, y) > 1$ and $\int_{D_2} f(x, y)d(x, y) > \int_{D_1} f(x, y)d(x, y)$.
    (c) $\int_{D_1} f(x, y)d(x, y) \leq 1$ and $\int_{D_2} f(x, y)d(x, y) \leq \int_{D_1} f(x, y)d(x, y)$.
    (d) $\int_{D_1} f(x, y)d(x, y) > 1$ and $\int_{D_2} f(x, y)d(x, y) \leq \int_{D_1} f(x, y)d(x, y)$.
    (e) None of the above statements holds true.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

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21. The equation

$$x^3y + \cos(xy) = 1$$

defines y as a differentiable function of x when (x, y) is near the point $(1, 0)$. Which of the following statements is true?

(a) $\left.\frac{dy}{dx}\right|_{(x,y)=(1,0)} \leq -2$.

(b) $-2 < \left.\frac{dy}{dx}\right|_{(x,y)=(1,0)} \leq -1$.

(c) $-1 < \left.\frac{dy}{dx}\right|_{(x,y)=(1,0)} \leq 0$.

(d) $0 < \left.\frac{dy}{dx}\right|_{(x,y)=(1,0)} \leq 1$.

(e) None of the above statements holds true.

22. Let $f(x, y) = \int_0^x u^y e^{-u} du$ for $x \in (-\infty, \infty)$, $y > 0$. Which of the following statements is true?

(a) $f_x(1, 1) > 1$.

(b) $f(-1, 1) \leq 1$.

(c) $\lim_{x\to\infty} f(x, 1) > 1$.

(d) $\lim_{x\to\infty} f(x, 2) = \infty$.

(e) None of the above statements holds true.

23. Let $D = \{(x, y) : x > 0, y > 0, 0 < x^2 + y^2 \leq 1\}$. Which of the following statements is true?

(a) $\int_D (x^2 + y^2) d(x, y) > 1$.

(b) $\int_D 2 d(x, y) \leq 1$.

(c) $\int_D (x + y) d(x, y) > 1$.

(d) $\int_D xy d(x, y) \leq 1$.

(e) None of the above statements holds true.

24. Let $S_n = \sum_{k=1}^{n} \sin\left(\frac{k}{n}\right)$. Which of the following statements is true?

(a) $\lim_{n\to\infty} S_n$ exists and $\lim_{n\to\infty} S_n < 1$.

(b) $\lim_{n\to\infty} \sqrt{S_n}$ exists and $\lim_{n\to\infty} \sqrt{S_n} < 1$.

(c) $\lim_{n\to\infty} S_n/n$ exists and $\lim_{n\to\infty} S_n/n > 1$.

(d) $\lim_{n\to\infty} S_n/(n^2)$ exists and $\lim_{n\to\infty} S_n/(n^2) < 1$.

(e) None of the above statements holds true.

25. Let $a_n = \int_0^\pi x^n \cos(x) dx$ for $n \geq 1$. Which of the following statements is true?

(a) $a_1 > 0$.

(b) $a_2 > 0$.

(c) $a_3 > 0$.

(d) $a_4 > 0$.

(e) None of the above statements holds true.

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