國立成功大學 107 學年度碩士班招生考試試題

系　所：太空與電漿科學研究所

考試科目：電磁學

考試日期：0205，節次：1

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

- Calculation processes have to be shown.
- System of unit used in the problems of this examination is the SI unit.
- The characters shown below are used to represent the physical quantities:
  Current density: $\boldsymbol{j}$, angular frequency: ω, magnetic field: $\boldsymbol{B}$, electric field: $\boldsymbol{E}$.
- Characters representing physical constants listed up below are available if necessary:
  Elementary charge: e [C], permittivity of vacuum: ε_0 [m^{-3} kg^{-1} s^4 A^2](or [$F \cdot m^{-1}$]), permeability of vacuum: μ_0 [m kg s^{-2} A^{-2}] (or [$H \cdot m^{-1}$]), speed of light in vacuum: c [m s^{-1}].

I. Electrostatic field (20 %)

i. A charged sphere S with uniform charge density ρ is located at $r = 0$, whose radius is R. Find the radial profile of the electric field $\boldsymbol{E}(r)$, for $0 < r \leqq R$ and $r \geqq R$. (5 %)

ii. Calculate the radial profile of the electrostatic potential $\phi(r)$, for $0 < r \leqq R$ and $r \geqq R$. The boundary condition is given as $\phi \longrightarrow 0$ at $r \longrightarrow \infty$. (5 %)

iii. Find the electrostatic energy U of the sphere S. (10 %)

II. Electric field in dielectric medium: (5 % each, total 25 %)

There is a point external charge Q at the center of a dielectric sphere, whose radius and dielectric constant (uniform in space inside the sphere) are a and ε, respectively (Fig. 1). Find the radial profiles of the electric flux density $\boldsymbol{D}$, the electric field $\boldsymbol{E}$, the electrostatic potential ϕ, the polarization charge density ρ_p, and the polarization surface charge density σ_p, respectively. The boundary condition is give as $\phi \longrightarrow 0$ at $r \longrightarrow \infty$. (Hint: the electric polarization $\boldsymbol{P}$ is expressed in a form $\boldsymbol{P} = P(r)\boldsymbol{r}/r$. Use the identity $\nabla \cdot (\boldsymbol{r}/|\boldsymbol{r}|^3) = 4\pi\delta(\boldsymbol{r})$, where $\delta(\boldsymbol{r})$ is the Dirac delta function.)

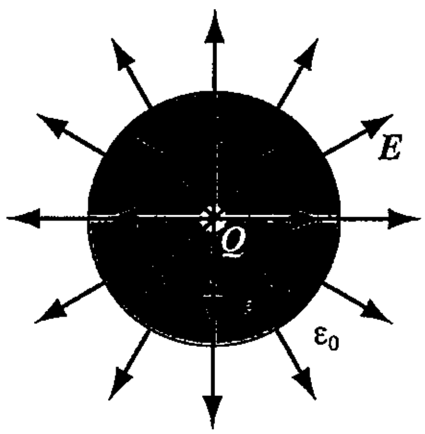

Fig. 1

III. Radiation from harmonic oscillation of an electric dipole: (25 %)

There is an oscillating electric dipole at the origin of a Cartesian coordinate (x, y, z), that can be expressed as an oscillation of electric current $\boldsymbol{I} = \boldsymbol{z}\, I_o\cos(\omega t)$, where $\boldsymbol{z}$, I_o, ω and t are the unit vector in the z direction, the amplitude of the current, the angular frequency and time, respectively. In the long-wavelength and far-field limit in which the length l of the harmonic oscillator is much shorter than the wavelength of the radiated electromagnetic waves and r >> (the wavelength), the vector potential $\boldsymbol{A}$ and the electrostatic potential ϕ are given as

$$\boldsymbol{A} = \boldsymbol{z}\,\frac{\mu_0}{4\pi r} I_0 l \cos\left[\omega\left(t - \frac{r}{c}\right)\right],$$

$$\phi = \frac{c\mu_0}{4\pi} I_0 l \frac{z}{r^2} \cos\left[\omega\left(t - \frac{r}{c}\right)\right].$$

Here, $r = (x^2+y^2+z^2)^{1/2}$.

(i) Find the r, θ, and φ components of the electric field $\boldsymbol{E}$ in the spherical coordinate shown in Fig. 2 by calculating the r, θ, and φ components of $\boldsymbol{A}$. Retain only terms proportional to r^{-1} by neglecting terms proportional to $r^{-1/2}$. (5 % × 3 = 15 %)

(ii) Find the radiation power per unit solid angle $P \equiv r^2|\overline{\boldsymbol{S}}|$ as a function of θ and draw the P vs θ graph, where $\overline{\boldsymbol{S}} \equiv \frac{1}{T}\int_0^T \frac{\boldsymbol{E}\times\boldsymbol{B}}{\mu_0} dt$ is the time-averaged Poynting vector, where T is one period of the oscillation. The magnetic field $\boldsymbol{B}$ is given as $\boldsymbol{B} = -\boldsymbol{\varphi}\frac{\mu_0}{4\pi r} I_0 l \frac{\omega}{c} \sin\theta \sin\left[\omega\left(t - \frac{r}{c}\right)\right]$. (9 % + 1 % = 10 %)

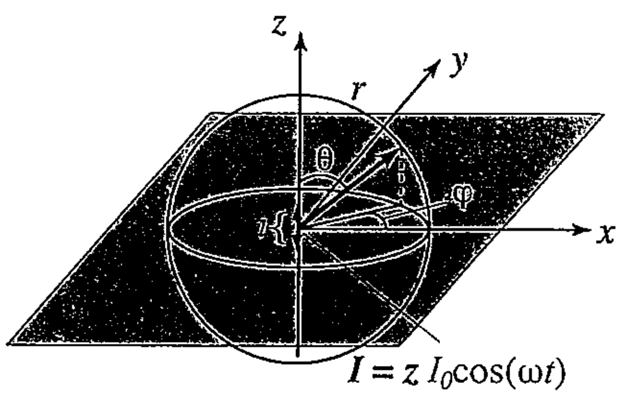

Fig. 2

IV. Derivation of the energy conservation equation of electromagnetic field in vacuum from Maxwell Eqs. (30 %)

i. Derive the following equation from Maxwell equations. The calculation process has to be clearly shown. Use the vector identity, $\nabla\cdot(\boldsymbol{A}\times\boldsymbol{B}) = \boldsymbol{B}\cdot(\nabla\times\boldsymbol{A}) - \boldsymbol{A}\cdot(\nabla\times\boldsymbol{B})$. (15 %)

$$\frac{1}{2}\frac{\partial}{\partial t}\left[\varepsilon_0\left|\vec{E}\right|^2+\frac{\left|\vec{B}\right|^2}{\mu_0}\right]+\nabla\cdot\left[\frac{\vec{E}\times\vec{B}}{\mu_0}\right]=-\vec{j}\cdot\vec{E}.$$

ii. Explain physical meaning of each term of the equation derived in the above question i **in English.** (5% x 3 = 15 %)