



- Find the solution of the equation $3x(xy - 2)dx + (x^3 + 2y)dy = 0$. (10 分)
- Find the solution of the equation $y'' - 2y' - 3y = 3t^2 + 4t - 5$ with $y(0) = 9$ and $y'(0) = -4$. (15 分)
- Find the solution of the system $\begin{cases} x_1' = 5x_1 - 3x_2 + 8 \\ x_2' = x_1 + x_2 + 32t \end{cases}$ with $\begin{cases} x_1(0) = 2 \\ x_2(0) = 0 \end{cases}$. (15 分)
- Find the solution of the equation $y'' - 3y' + 2y = 4e^{2t}$ with $y(0) = -3$ and $y'(0) = 5$. (10 分)

(a) Prove that the Laplace transform of $4e^{2t}$ is $\frac{4}{s-2}$.

(b) Solving the differential equation by using Laplace transform.

- Perform the indicated operation, give that (10 分)

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 2 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 3 & 0 \\ 0 & -2 & 4 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 \\ 4 & 3 \\ 0 & -1 \end{bmatrix}$$

(a) $(A+B)C$ (b) If $2X - 6(2A - B) = 0$, Find X.

- Find the eigenvalues and eigenvectors of A. (10 分)

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & 0 & 3 \end{bmatrix}$$

- (a) A real square matrix $A = [a_{jk}]$. Define the following terms: (using by A^T , A , A^{-1})
 (1) Symmetric matrix (2) Skew-Symmetric matrix (3) Orthogonal matrix (6 分)

(b) Determine the nature (Symmetric or Skew-Symmetric or Orthogonal) of the following matrices. (8 分)

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 9 & -12 \\ -9 & 0 & 20 \\ 12 & -20 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 1/7 & 3/7 & 2/7 \\ 3/7 & 2/7 & 5/7 \\ 2/7 & 5/7 & 3/7 \end{bmatrix}$$

- If $\vec{A} = 2\vec{i} + \vec{j} - \vec{k}$, $\vec{B} = \vec{i} - 3\vec{j} - 5\vec{k}$ Find (a) $\vec{A} \cdot \vec{B}$ (b) $\vec{A} \times \vec{B}$ (c) the $\cos\theta$ between \vec{A} and \vec{B} (d) the projection of \vec{A} on \vec{B} . (16 分)