國立交通大學 107 學年度碩士班考試入學試題

科目：微積分(4711)　　考試日期:107 年 2 月 2 日 第 3 節

系所班別：生物科技學系　　組別：生科系丙組　　第 1 頁,共 2 頁

【不可使用計算機】＊作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符！！

1. **(30 %, 15% for each)** Van der Waals gases obey the following equation of state (van der Waals equation):

$$P = \frac{RT}{\bar{V} - b} - \frac{a}{\bar{V}^2}$$

Where a, and b are coefficients, R is gas constant ($8.3145\ \mathrm{J \cdot K^{-1} \cdot mol^{-1}}$), $\bar{V}$ is molar volume ($\frac{V}{n}$), T is temperature in unit of K and P is pressure.

(a) $\left(\frac{\partial U}{\partial V}\right)_T$ indicates how internal energy changes with the change of volume under constant temperature and it obeys the equation shown below:

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P$$

where P is pressure, T is temperature and V is volume. Show that $\left(\frac{\partial U}{\partial V}\right)_T = \frac{an^2}{V^2}$ for van der Waals gases.

(b) The change of the *internal energy U* could be represented as following:

$$dU = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV$$

Calculate ΔU_m for the isothermal reversible expansion of 1 mole argon (Van der Waals coefficients :

a = 1.337 atm $\mathrm{L^2\ mole^{-2}}$; b =3.20×10^{-2} L $\mathrm{mole^{-1}}$) for an initial volume of 1.00 L to 24.0 L at constant temperature of 298K.

2. **(20%)** The molar heat capacities $C_{p,m}$ (unit: $\mathrm{J \cdot K^{-1} \cdot mole^{-1}}$) of N_2 at constant pressure obeys the following relation with temperature: $C_{p,m} = a + bT + \frac{c}{T^2}$ where T is temperature and $a = 28.58\ \mathrm{J \cdot K^{-1} \cdot mole^{-1}}, b = 3.77 \times 10^{-3}\ \mathrm{J \cdot K^{-2} \cdot mole^{-1}}$ *and* $c = -0.5 \times 10^5\ \mathrm{J \cdot K \cdot mole^{-1}}$

The change of molar entropy due to the change of temperature under constant pressure is as following :

$$\Delta S_m = \int_{T_i}^{T_f} \frac{C_{p,m}}{T} dT$$

What is the change of molar entropy of N_2 when it is heat from 298K to373K.

3. To calculate

(a) (5%) $\lim_{x\to 0}\frac{2\tan^2 x}{1-\cos^2 x}$

(b) (5%) $\lim_{n\to\infty}(\sqrt{n^2+n}-n)$

(c) (5%) $\lim_{x\to+\infty}\frac{\ln(\ln x)}{\ln(x-\ln x)}$

4. To calculate

(a) (5%) $\int_0^{\frac{3\pi}{2}}|\sin x|\,dx$

(b) (5%) $\int e^{-\sqrt{x}}\,dx$

(c) (5%) $\int_0^{\pi}\int_x^{\pi}\frac{\sin y}{y}\,dy\,dx$

(d) (5%) $\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\sqrt{9-x^2-y^2}\,dy\,dx$

5. To calculate the following statements.

(a) (5%)Determine the area of the region enclosed by $y=2x^2$ and $y=-3x^2+5$

(b) (5%)Let A be the area found in question a. Determine the volume of the solid obtained by rotating area A around x-axis

(c) (5%)Determine the volume of the solid obtained by rotating area A around y-axis