



1. A real sequence $\{x_n\}$ satisfies $7x_{n+1} = x_n^3 + 6$ for $n \geq 1$.
If $x_1 = \frac{1}{2}$, find its limit. (10 分)
2. The formula in the Mean-Value Theorem can be written as follow:
$$\frac{f(x+h) - f(x)}{h} = f'(x+\theta h),$$
 where $0 < \theta < 1$. Determine θ as function of x and h when $f(x) = x^2$. (10 分)
3. Find the volume of the region bounded by the parabolic cylinder $z = 4 - x^2$ and the planes $x = 0$, $y = 0$, $y = 6$, $z = 0$. (5 分)
4. Evaluate $\int_{-1}^1 \frac{dx}{\sqrt[3]{x+1}}$. (5 分)
5. Let $S_n = nxe^{-nx^2}$, $n = 1, 2, 3, \dots$, $0 \leq x \leq 1$. Determine whether
$$\lim_{n \rightarrow \infty} \int_0^1 S_n(x) dx = \int_0^1 \lim_{n \rightarrow \infty} S_n(x) dx.$$
 (5 分)
6. For what values of x does $\sum_{n=1}^{\infty} \frac{1}{2n-1} \left(\frac{x+2}{x-1}\right)^n$ converge? (5 分)
7. If $y = x^x$, compute $\frac{dy}{dx}$. (5 分)
8. Evaluate $\int_0^{\infty} \frac{\sin^3 x}{x}$. (5 分)
9. Find the indefinite integral $\int \frac{e^x + e^{-x}}{2} \cos x dx$. (10 分)
10. Compute $\sum_{x=0}^{\infty} \frac{1}{2^x x!}$. (10 分)
11. Evaluate $\int_0^{\infty} e^{-x^2} dx$. (10 分)
12. Find the equation of the tangent line to the graph $y^2 - x^2 y - x^3 - 1 = 0$ at the point $(x_0, y_0) = (1, 2)$. (10 分)
13. Evaluate the summation $\sum_{n=1}^{\infty} n^2 \left(\frac{1}{3}\right)^n$. (10 分)