國立雲林科技大學
100 學年度碩士班暨碩士在職專班招生考試試題

系所：財金系
科目：統計學(4)

1. (10 points) A local bank reviewed its credit card policy with the intention of recalling some of its credit cards. In the past, approximately 5% of cardholders defaulted, leaving the bank unable to collect the outstanding balance. Hence, management established prior probability of 0.05 that any particular cardholder will default. The bank also found that the probability of missing a monthly payment is 0.20 for customers who do not default. Of course, the probability of missing a monthly payment for those who default is 1.

   Given that a customer missed one or more monthly payments, compute the posterior probability that the customer will default.

2. (10 points) A shipment of 10 items has two defective and eight non-defective items. In the inspection of the shipment, a sample of items will be selected and tested. If a defective item is found, the shipment of 10 items will be rejected.

   If a sample of three items is selected, what is the probability that the shipment will be rejected?

3. (10 points) If the joint probability density of X and Y is given by

$$f(x,y)=\begin{cases} \frac{2}{3}(x+2y) & \text{for } 0<x<1,\ 0<y<1 \\ 0 & \text{elsewhere} \end{cases}$$

   find the conditional variance of X given $Y=\frac{1}{2}$.

4. (10 points) If the joint density of X_1 and X_2 is given by

$$f(x_1,x_2)=\begin{cases} 6\cdot e^{-3x_1-2x_2} & \text{for } x_1>0,\ x_2>0 \\ 0 & \text{elsewhere} \end{cases}$$

   find the probability density of $Y= X_1 + X_2$.

5. (10 points) If X has the exponential distribution given by

$$f(x)=\begin{cases} e^{-x} & \text{for } x>0 \\ 0 & \text{elsewhere} \end{cases}$$

   find the probability density of the random variable $Y=\sqrt{X}$.

6. (15 points)

(a) Suppose that a simple regression has quantities $\sum(y_i-\bar{y})^2=631.63$ and $\sum\hat{e}_i^2=182.85$, find R^2.

(b) Suppose that a simple regression has quantities N=20, $\sum y_i^2=5930.94$, $\bar{y}=16.035$, and SSR=666.72, find R^2.

(c) Suppose that a simple regression has quantities R^2=0.7911, SST=552,36, and N=20, find $\hat{\sigma}^2$.

7. (10 points) Suppose our estimate model is $y_i=\beta x_i+u_i$, i=1,...,n, However, the real model is $y_i=\alpha+\beta_0 x_i+v_i$, where $E(v_i\mid x)=0$ and $Var(v_i\mid x)=\sigma_0^2$. Find the variance of OLS estimator of β.

8. (5 points) Which of the following model(s) will result in perfect multicollinerity problem?

(a) $y=\alpha+\beta x+\gamma(1/x)+u$

(b) $y=\alpha+\beta x+\gamma(x^2)+u$

(c) $y=\alpha+\beta(\ln x)+\gamma(\ln x^2)+u$

9. (10 points) Suppose there are two factors A and B. Each factor has four categories. Please finish the following table

Source of variance	sum of square	degrees of freedom	average	F value
A	60.2	____	____	____
B	12.3	____	____	____
Cross	____	____	4.1	____
Error	____	____	____	
Total	237.8	____		

10. (10 points) Let $\hat{\vartheta}$ is an unbiased estimator of ϑ_0, Z is a random variable that mean and variance are all equal to one, and $cov(\hat{\theta},Z)$=2. Find the Variance and mean square error (MSE) of $\tilde{\theta}=\hat{\theta}-Z$.