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100 學年度碩士班暨碩士在職專班招生考試試題

科目：工程數學(3)

1. In problems (a)~(c), please solve for $y = y(x)$ (15 分)

   (a) $y'' - 12y = 0$

   (b) $y'' - \dfrac{2}{x+1}y' + \dfrac{2}{(x+1)^2}y = 0$

   (c) $xy'' + (1 - 2x)y' + (x - 1)y = 0$

2. A function $y = y(x)$ is the 1st order differential equation: (10 分)

   $$\frac{dy}{dx} = -\frac{3x^2y + 6xy + y^2/2}{3x^2 + y}$$

   (a) Does the differential equation satisfy the "Condition of Exactness"?
   (b) Solve the differential equation using the method of integration factor.

3. A Bernoulli's differential equation $\dfrac{dy}{dx} + p(x)y = g(x)y^a$ have the value of a is any real number but not equal to 0 or 1.(15 分)
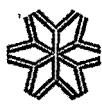
   (a) Set $u(x) = [y(x)]^{1-a}$ and show that the above differential equation can be transformed into a linear form. (5 分)

   (b) Use the result of (a) to solve : $\dfrac{dy}{dx} + \dfrac{y}{x} = -2xy^2$ (10 分)

4. Use Laplace transforms to solve the equation system in the initial condition of $x(0) = 2, y(0) = 0$.(10 分)

   $$\begin{cases} x' + 3x - y = 2 \\ x' + y' + 3x = 0 \end{cases}$$

5. Find the inverse of $A = \begin{pmatrix} 2 & 0 & 1 \\ -2 & 3 & 4 \\ -5 & 5 & 6 \end{pmatrix}$.(15 分)

6. Find the eigenvalues and eigenvectors of $A = \begin{pmatrix} 9 & 1 & 1 \\ 1 & 9 & 1 \\ 1 & 1 & 9 \end{pmatrix}$.(15 分)

7. Find the directional derivative of $F(x,y,z) = xy^2 - 4x^2y + z^2$ at $(1,-1,2)$ in the direction of $6i+2j+3k$.(10 分)

8. If $F = (x^2y^3 - z^4)i + 4x^5y^2zj - y^4z^6k$, find (a) curl F (b) div F (c) div (curl F).(10 分)