



本試題共 8 題，共計 100 分，請依題號作答並將答案寫在答案卷上，違者不予計分。

1. Solve the general solution of differential equations: (本題共 20 分)

$$(1) \frac{dy}{dx} = 2x \quad (5 \text{ 分})$$

$$(2) x \sin(y) \frac{dy}{dx} = \cos(y) \quad (5 \text{ 分})$$

$$(3) [(x-1)^3 D^3 + (x-1)^2 D^2 - 4(x-1)D] y = 3(x-1)^2 \quad [\text{Note: } D^n y = y^{(n)} = \frac{d^n}{dx^n} y] \quad (10 \text{ 分})$$

2. Find the Laplace transform of the function: $f(t) = te^{-3t} \sin(2t)$. (本題 10 分)

3. Find the inverse Laplace transform of the function: $F(s) = 2 \ln \left[\frac{(s-1)}{(s+1)} \right]$. (本題 10 分)

4. Find the eigenvalues and eigenvectors of the matrix A and A^{-1} (inverse of matrix A),

$$\text{where } A = \begin{pmatrix} -2 & 0 \\ 1 & 4 \end{pmatrix}. \quad (\text{本題 } 10 \text{ 分})$$

5. Find the Fourier half cosine and Fourier half sine expansions (i.e. Fourier cosine series and Fourier sine series) of $f(x)$ for

$$f(x) = \begin{cases} 1, & 0 \leq x < 1 \\ 0, & 1 \leq x \leq 3 \\ -1, & 3 < x \leq 5 \end{cases} \quad (\text{本題 } 15 \text{ 分})$$

6. Find the Fourier integral for the function

$$f(x) = \begin{cases} -1, & -\pi \leq x < 0 \\ 1, & 0 < x \leq \pi \\ 0, & |x| > \pi \end{cases} \quad (\text{本題 } 10 \text{ 分})$$

7. Find values of a , b , c and d such that the following system of linear equations has

(i) exactly one solution, (ii) no solution and (iii) an infinite number of solutions. (本題 15 分)

$$x + y = 2$$

$$y + z = 2$$

$$x - z = d$$

$$ax + by + cz = 0$$

8. Find all values of t for which the set S is linear independent.

$$S = \left\{ \begin{bmatrix} t \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ t \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ t \end{bmatrix} \right\} \quad (\text{本題 } 10 \text{ 分})$$