國立雲林科技大學
100學年度碩士班暨碩士在職專班招生考試試題

系所：資工系、電子光電所
科目：線性代數(2)

本試題共十題，共計 100 分，請依題號作答並將答案寫在答案卷上，違者不予計分。

1. (5%；複選全對才給分) A and B are 3X3 matrices and $|A| = -3$, $|B|=2$. Which statements are correct?
   (a) $|AB|=-6$ ；(b) $|2AB^{-1}|=-6$ ；( c) $|(A^2)^t|=-9$ ；(d) $|(A^t)^2|=9$ ；(e ) $|(A^2B^{-1})^t|=-18$

2. (10%) Consider the two vectors, (1, 2, -1) and (3, 1, 0). (a)(2%) Find the norms of the two vectors. (b) (2%) Normalize the two vectors. (c) (6%) Find a vector that is orthogonal to the two vectors.

3. (15%) Consider the matrix $A=\begin{bmatrix} 9 & -3 & 3 \\ -3 & 6 & -6 \\ 3 & -6 & 6 \end{bmatrix}$.

   (a) (5%)Find its eigenvalues. (b) (5%) Find the corresponding normalized eigenvectors.
   (c) (5%) Find the matrix $A^{10}$.

4. (10%) Asus and Acer are competing for customers at notebook market. A study has been made of customer satisfaction with the various companies. The results are expressed by the following matrix R. The First column of R implies that 75% of those currently using Asus notebook are satisfied and intend to use Asus next time, while 25% of those using Asus are dissatisfied and plan to use Acer next time. There is a similar interpretation to the second column of R. If the current trends continue, how will the customer distribution eventually settle?

   (from)

   $$R = \begin{array}{c} \text{Asus} \quad \text{Acer} \\ \begin{bmatrix} 75\% & 20\% \\ 25\% & 80\% \end{bmatrix} \end{array} \begin{array}{l} \text{Asus} \\ \text{Acer} \end{array}$$

5. (5%) Determine the inverse of the matrix $\begin{bmatrix} 5 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 2 & 3 \end{bmatrix}$, if it is exists, using the method of Gauss-Jordan elimination.

6. (5%) Determine the equation of the polynomial of degree two whose graph passes through the point (1, 6), (2, 3), (3, 2)

7. (15%) Determine the inverse of each of the following matrices, if it exists, using the method of Gauss-Jordan elimination.

(a) (5%) $\begin{bmatrix} 1 & 2 & -3 \\ 1 & -2 & 1 \\ 5 & -2 & -3 \end{bmatrix}$

(b) (5%) $\begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & -3 \\ 1 & -2 & 0 \end{bmatrix}$

(c) (5%) $\begin{bmatrix} -3 & -1 & 1 & -2 \\ -1 & 3 & 2 & 1 \\ 1 & 2 & 3 & -1 \\ -2 & 1 & -1 & -3 \end{bmatrix}$

8. (10%) Solve the following problems.

(a) (5%) Find $x$ such that $\begin{bmatrix} 2x & 7 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 2 & -7 \\ -1 & 4 \end{bmatrix}$.

(b) (5%) Find $A$ such that $(4A^t)^{-1} = \begin{bmatrix} 2 & 3 \\ -4 & -4 \end{bmatrix}$, where the superscript $t$ denotes the transpose operation.

9. (9%) Prove that the transformation $T$: $\mathbf{R}^2 \to \mathbf{R}^2$ defined by $T(x,y) = (3x, x+y)$ is linear. Find the images of the vectors $(1,3)$ and $(-1,2)$ under this transformation.

10. (16%) Consider the linear transformation $T$ defined by each of the following matrices. Determine the kernel and range of each transformation. Show that $\dim\ker(T) + \dim\text{range}(T) = \dim\text{domain}(T)$ for ea ch transformation. (Note that the abbreviations of dim and ker denote dimension and kernel, respectively.

(a) (8%) $\begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$

(b) (8%) $\begin{bmatrix} 1 & 1 & 5 \\ 0 & 1 & 3 \\ 2 & 1 & 7 \end{bmatrix}$