



1. Given the following **undirected** graph. (20%)

$$G=(N,A), N(G)=\{a, b, c, d, e\},$$

$$A(G)=\{(a, b), (a, c), (b, c), (b, d), (b,e), (c, d), (c, e), (d, e)\}$$

(a) Complete the graph. (4%)

(b) Determine the adjacency matrix of the graph. (5%)

(c) Determine the number of spanning trees of the graph. (11%)

2. Solve the following recurrence equation by giving the tightest (up to a constant factor) upper bound for $T(n)$ in Big-Oh notation. Assume that $T(n) = c$, for $n \leq 1$ and c is a constant for the following recurrence.

$$(Eq) T(n) = 4T\left(\frac{n}{4}\right) + n \log n \quad (20\%)$$

3. Determine the best "big Oh" of time complexity for each following expression. (10%)

$$(1) a = 5 + 10 + 15 + \dots + 5n$$

$$(2) b = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$$

$$(3) c = \frac{(n^2 + \log n)(n+9)}{n+n^2}$$

$$(4) d = 2 \log n - 8n + n \log n$$

4. Determine if the following are statements? (10%)

(a) The moon is made of green cheese.

(b) He is certainly a tall man.

(c) Two is a prime number.

(d) Will the game be over soon?

(e) Next year interest rates will rise.

5. Prove that the amount of postage greater than or equal to 8 cents can be built using only 3-cent and 5-cent stamps. (10%)

6. The following algorithm is a recursive version of the sequential search algorithm: (10%)

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SequentialSearchRecursive(list L; integer i, n; itemtype x)
//searches list L from L[i] to L[n] for item x
  if i > n then
    write("Not found")
  else
    if L[i] = x then
      write("Found")
    else
      SequentialSearch(L, i + 1, n, x)
    end if
  end if

```

Please analyze the algorithm using recurrence relations.

7. For the relation $\{(1,1), (2,2), (1,2), (2,1), (1,3), (3,1), (3,2), (2,3), (3,3), (4,4), (5,5), (4,5), (5,4)\}$, what is [3] and [4]? (10%)

8. Write $\text{gcd}(1326, 252)$ as a linear combination of 1326 and 252. (10%)