

國立中央大學 107 學年度碩士班考試入學試題

所別： 通訊工程學系碩士班 不分組(一般生)

共 2 頁 第 1 頁

科目： 工程數學(線性代數、機率)

本科考試禁用計算器

*請在答案卷(卡)內作答

1. (15%) Let A be an $m \times n$ matrix.
 - (1) (5%) Show that if B is a nonsingular $m \times m$ matrix, then BA and A have the same null space and hence the same rank.
 - (2) (5%) If $m = n$, and let L_A be the linear operator defined by $L_A(\mathbf{x}) = A\mathbf{x}$. Show that L_A maps \mathbb{R}^n onto the column space of A .
 - (3) (5%) If A is an orthogonal matrix (also $m = n$), use mathematical induction to prove $\|A^m \mathbf{x}\| = \|\mathbf{x}\|$ for any $\mathbf{x} \in \mathbb{R}^n$.

2. (10%) Let

$$A = \begin{bmatrix} 1 & 0 & 1 & 2 & 1 & 2 \\ -2 & 3 & -1 & -8 & -6 & -3 \\ 4 & 4 & 6 & 6 & 2 & 10 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}.$$

- (1) (5%) Find all solutions to the system $A\mathbf{x} = b$.
 - (2) (5%) Find the bases for the column space of A and the null space of A .
3. (10%) Consider the data points $(1,1)$, $(4,-1)$, $(5,3)$.
 - (1) (5%) Find the linear function $y = c_0 + c_1x$ that gives the best least square fit to the points.
 - (2) (5%) Find a quadratic polynomial of the form $y = c_0 + c_1x + c_2x^2$ that passes through all three of the data points.

4. (15%) Let A be the matrix

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 5 & 0 \\ -2 & 0 & 4 \end{bmatrix}.$$

- (1) (5%) Find the eigenvalues of A .
 - (2) (5%) Find the eigenvectors of A .
 - (3) (5%) For any integer t , write a formula for A^t .

參考用

注意:背面有試題

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5. (20%) Discrete random variable X has probability mass function

$$P_X(x) = \begin{cases} 0.2 & x = -1, \\ 0.4 & x = 0, \\ 0.4 & x = 1, \\ 0 & \text{otherwise.} \end{cases}$$

Let B denote the event that $X \geq 0$.

- (1) (10%) Find conditional probability mass function of X given the event B .
 - (2) (10%) Find the conditional variance of X given the event B .
6. (10%) Continuous random variables X and Y have joint probability density function

$$f_{X,Y}(x,y) = \begin{cases} 6xy^2 & 0 \leq x \leq 1, 0 \leq y \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

Let random variable $Z = \max(X, Y)$. Find the expected value of Z .

7. (20%) Let random variable $W = X_1 + \cdots + X_{100}$ be the sum of 100 i.i.d. continuous uniform random variables, each with expected value 10 and variance 4. Random variable $U = 5 + 0.1W$.
- (1) (10%) Find the expected value of U .
 - (2) (10%) Find the variance of U .

注意:背面有試題

參考用