1. (30%)

By definition, the function $u(t-a)$ is 0 for $t < a$, has a jump of size 1 at $t = a$, and is 1 for $t > a$.

Please find the Laplace transform for the function shown below

$f(t) = sin(2t)u(t-1)$

2. (20%)

Given an equation as below

$y'' + 3y' + 2y = 2f(t)$,

with $y(0)=1.5$, and a force function $f(t)$ given in figure 1,

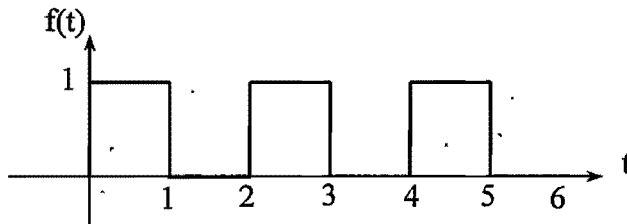please find $y(t)$ **in the range $0 \leq t \leq 1$. (Explicit form is required)**

Figure 1.

3. Let $A = \begin{bmatrix} 5 & 0 & 0 \\ 1 & 0 & 3 \\ 0 & 0 & -2 \end{bmatrix}$

Find (1) the eigenvalues, (2) the corresponding eigenvectors of the matrix A, and (3) the matrix P, matrix P^{-1} and the diagonal matrix D such that $P^{-1}AP = D$. (25%)

4. Let $\vec{f} = f_1 \vec{i} + f_2 \vec{j} + f_3 \vec{k}$ be a vector field where $f_1 = f_1(x, y, z)$, $f_2 = f_2(x, y, z)$, and $f_3 = f_3(x, y, z)$, and $\phi = \phi(x, y, z)$ be a scalar field. Prove that (1) div (curl $\vec{f}$) = 0 (10%) and (2) $\nabla \cdot (\phi \vec{f}) = \vec{f} \cdot \nabla \phi + \phi \nabla \cdot \vec{f}$ (15%)。