

國立高雄應用科技大學
100 學年度碩士班招生考試
金融資訊研究所

准考證號碼 (考生必須填寫)

統計學

試題 共 2 頁，第 1 頁

- 注意：a. 本試題共 6 題，每題之配分均註明於題目中，共 100 分。
b. 作答時不必抄題。
c. 請詳細寫出求解與證明過程，否則不予計分。
d. 考生作答前請詳閱答案卷之考生注意事項。

- Let X_1, \dots, X_{25} be a random sample from $N(\mu_X, 25)$ and Y_1, \dots, Y_9 be a random sample from $N(\mu_Y, 36)$. These samples are assumed to be independent and let \bar{X} and \bar{Y} be their sample average respectively.
 - What's the distribution of $2\bar{X} + \bar{Y}$? (10%)
 - For testing the hypothesis $\mu_X - \mu_Y = 4$ versus the alternative $\mu_X - \mu_Y \neq 4$, what test statistic will you use? What's the exact distribution of this test statistic? (10%)
- Prove the following statements: If X_1 and X_2 are two random variables, then $E(X_2) = E[E(X_2 | X_1)]$, where $E(X_2 | X_1)$ is the conditional expectation of X_2 given X_1 . [Hint: This is the law of iterated expectation] (10%)
 - Suppose that X_1, X_2, \dots are i.i.d. Exponential random variables with mean μ . Let N be a Poisson random variable (independent of the X_i 's) with mean λ . Define the random variable $Y = \sum_{i=1}^N X_i$ where $Y = 0$ if $N = 0$. Find $E(Y)$. (10%)

3. Let X_1, \dots, X_n be a random sample from a Poisson distribution with parameter λ and let $\hat{\lambda} = n^{-1} \sum_{i=1}^n X_i$ be an estimator of λ . Find the bias, standard error, and mean squared error of this estimator. (15%)

4. Let X_1, \dots, X_n be a random sample from Uniform distribution $U(0, \theta)$, $\theta > 0$.
 (1) Find the method of moment estimator $\hat{\theta}$ of θ . [Hint: Use the first moment.] (5%)

(2) Prove that $\hat{\theta}$ is an unbiased and consistent estimator of θ . (10%)

5. Prove the following statement. (20%)

Let X and Y be two random variables. The variance of X can be decomposed as

$$\text{var}(X) = E[\text{var}(X | Y)] + \text{var}[E(X | Y)]$$

6. John estimates the following regression model (the figures in the parenthesis are the standard error of the parameter estimates)

$$\hat{y}_i = 0.683 + 0.402x_{2i} - 0.891x_{3i}, \quad R^2 = 0.96$$

(0.436) (0.291) (0.763)

By considering the t -ratio and the value of R^2 , Mary thinks that there may be a serious problem in such regression. What the problem might be? How might you go about solving the perceived problem? (10%)