

國立高雄應用科技大學
100 學年度碩士班招生考試
機械與精密工程研究所(丙、丁組)

准考證號碼 (考生必須填寫)

工程數學

試題 共 2 頁，第 1 頁

注意：a. 本試題共 8 題，共 200 分。

b. 作答時不必抄題。

c. 考生作答前請詳閱答案卷之考生注意事項。

1. Find the general solution of the differential equation. (10%)

$$y' + ky = e^{-kx}$$

2. Solve the initial value problem. (20%)

$$x^2 y'' + xy' - 4y = 0; y(1) = 7, y'(1) = -3$$

3. (a) Let $L\{f(t)\} = F(s)$ denote the Laplace transform of the function $f(x)$. Show that $L\{\delta(t-a)\} = e^{-as}$. (10%)

(b) Use the Laplace transform to solve the initial value problem. (20%)

$$y'' + 4y' + 5y = \delta(t-1), \quad y(0) = 0, y'(0) = 3$$

4. Consider a system of the first-order differential equations

$$X' = AX$$

where

$$A = \begin{pmatrix} 5 & 3 \\ 1 & 3 \end{pmatrix}, \quad X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

(a) Find the eigen values and its corresponding eigen vectors of the system. (20%)

(b) Find the general solution of the system. (20%)

5. Compute the derivative of $\frac{d}{dt} [\vec{F}(t) \times \vec{G}(t)]$. (10%)

$$\vec{F}(t) = -9\vec{i} + t^2\vec{j} + t^2\vec{k}, \quad \vec{G}(t) = e^t\vec{i}$$

見背面

6. Evaluate $\iint_S \vec{F} \cdot \vec{N} dA$ over the hemisphere $x^2 + y^2 + z^2 = 1, z \geq 0$, including the base consisting of point (x, y) with $x^2 + y^2 \leq 1$. (40%)

$$\vec{F} = 4x\vec{i} - z\vec{j} + x\vec{k}$$

7. Find the Fourier series of the function on the interval. (30%)

$$f(x) = \begin{cases} 1 & \text{for } -\pi \leq x < 0 \\ 2 & \text{for } 0 \leq x \leq \pi \end{cases}$$

8. (a) Write the following complex number in polar form. (10%)

$$-2 + 2i$$

- (b) Find the value of the derivative of the following complex number. (10%)

$$\frac{z-i}{z+i} \text{ at point } (0, i)$$