

國立嘉義大學 107 學年度
土木與水資源工程學系碩士班招生考試試題

科目：工程數學 (※禁止使用計算機)

1. Find the solution of the equation $y'' - 4y' + 4y = 0$ for which $y = 3$ and $y' = 4$ when $x = 0$. (25%)

2. Calculate the work done by the force $\vec{F} = y\vec{i} + 2x\vec{j}$ along the straight line from point $(0, 0)$ to point $(1, 1)$. (25%)

3. Given a matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and evaluate the following:

(a) Find the eigenvalues and eigenvectors of A . (10%)

(b) Find the eigenvalues and eigenvectors of A^{-1} . (5%)

(c) Find an orthogonal matrix P , that is $P^{-1} = P^T$, such that $P^{-1}AP = D$, where D is the diagonal matrix of the eigenvalues of A . (10%)

4. Given a function $f(x) = x + \pi$, $-\pi < x < \pi$ and evaluate the following:

(a) Find the Fourier series of f on the given interval. (15%)

(b) Use the result to show the sum of the series $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$. (10%)