

國立嘉義大學 107 學年度
應用數學系碩士班（甲組）招生考試試題

科目：微積分

說明：本考試試題為問答、計算、證明題，每題 10 分，請標明題號，同時將詳細過程作答在「答案卷」上。

1. Describe Part I and Part II of the Fundamental Theorem of Calculus in detail.
2. Show that $f(x) = \frac{|x^2-1|}{x-1}$ is not differentiable at $x=1$.
3. Evaluate $\frac{d}{dx} \left[\int_0^{x^3} \sqrt{\ln(t^2+3)} dt \right]$.
4. Find the sum of the series $\sum_{n=1}^{\infty} \frac{1}{4n^2-1}$.
5. Evaluate $\iint_R (2x+y) dA$, where $R = \{(x,y): 1 \leq x \leq 5, 2 \leq y \leq 6\}$.
6. Suppose that $4x^2 + 3\sin(x+y) + 6\ln(y^2+1) = 1$. Find $\frac{dy}{dx}$.
7. Find the arc length of the graph of $y = \frac{1}{6}x^3 + \frac{1}{2x}$ on the interval $\left[\frac{1}{2}, 2\right]$.
8. Decide whether the series $\sum_{n=1}^{\infty} \frac{e^{2n}}{n^n}$ converges or diverges.
9. If $F(x) = f(3f(4f(x)))$, where $f(0) = 0$ and $f'(0) = 2$, find $F'(0)$.
10. Let $f(x, y) = y^2 e^{\sin x} \ln(x+y)$. Find $\nabla f(x, y)$.