國立臺灣大學101學年度碩士班招生考試試題

※ 注意：請於試卷內之「非選擇題作答區」標明題號依序作答。

1-10 題為填充題

1. If $\begin{bmatrix}10 & 1\\ 19 & 2\end{bmatrix} = \begin{bmatrix}1 & 0\\ a & 1\end{bmatrix}\begin{bmatrix}1 & b\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ c & 1\end{bmatrix}$ and $a, b, c \in \mathbf{R}$, then $\begin{bmatrix}a\\ b\\ c\end{bmatrix} =$__________.(5%)

2. If $A = \begin{bmatrix}2 & 1 & d\\ 1 & 0 & -1\\ 3 & 1 & 2\end{bmatrix}$ and $\det(A) = 0$, then $d =$__________.(5%)

3. If $B = \begin{bmatrix}2 & 1 & 3 & 1 & 0 & 4\\ 1 & 2 & 3 & -1 & 1 & 5\\ 0 & 1 & 1 & -1 & 2 & 2\\ 1 & 1 & 2 & 0 & 2 & 3\end{bmatrix}$ then $\operatorname{rank}(B) =$__________.(5%)

4. $\begin{bmatrix}\frac{1}{2} & \frac{\sqrt{3}}{2}\\ -\frac{\sqrt{3}}{2} & \frac{1}{2}\end{bmatrix}^{300} =$__________.(5%)

5. Suppose $A \in \mathbf{R}^{3\times 3}$ and $\det(xI_3 - A) = x^3 - x^2 + 3x - 2$, then $\det(xI_3 - A^2) =$__________.(5%)

6. Let $A \in \mathbf{R}^{3\times 3}$ and $P = \begin{bmatrix}1 & 1 & 1\\ 2 & 3 & 3\\ 1 & 3 & 4\end{bmatrix}$. If $AP = P\begin{bmatrix}2 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & -1\end{bmatrix}$ and $A^T\begin{bmatrix}x\\ y\\ 1\end{bmatrix} = \begin{bmatrix}x\\ y\\ 1\end{bmatrix}$

   then $\begin{bmatrix}x\\ y\end{bmatrix} =$__________.(5%)

7. Let $A = \begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}$ and $B = \begin{bmatrix}2 & 13\\ -1 & 6\end{bmatrix}$. If $AC + CA = B$ then the matrix $C =$__________.(5%)

8. Let $H = \begin{bmatrix}1 & 2 & 3 & 4\\ 5 & 6 & 7 & 8\\ 9 & 10 & 11 & 12\\ 13 & 14 & 15 & 16\end{bmatrix}$. Suppose $H^3 = \alpha H^2 + \beta H + \gamma I_4$, $\alpha, \beta, \gamma \in \mathbf{R}$, then

   $\begin{bmatrix}\alpha\\ \beta\\ \gamma\end{bmatrix} =$__________.(5%)

9. If $T = \begin{bmatrix}1 & 2 & 2^2 & 2^3 & 2^4 & 2^5\\ 2 & 1 & 2 & 2^2 & 2^3 & 2^4\\ 2^2 & 2 & 1 & 2 & 2^2 & 2^3\\ 2^3 & 2^2 & 2 & 1 & 2 & 2^2\\ 2^4 & 2^3 & 2^2 & 2 & 1 & 2\\ 2^5 & 2^4 & 2^3 & 2^2 & 2 & 1\end{bmatrix}$ then $\det(T) =$__________.(5%)

10 Let $S = \{\sum_{k=1}^{99} x_k x_{k+1} \mid x_1, x_2, \cdots, x_{100} \in \mathbf{R}$ and $x_1^2 + x_2^2 + \cdots + x_{100}^2 = 1\}$ then the largest number in S is__________.(5%)

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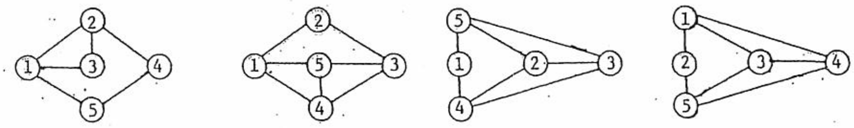
11 Determine which pairs of the following graphs are isomorphic. Also give an isomorphism for each isomorphic pair. (10%)



(a) (b) (c) (d)

12 For $n \geq 1$, the nth *triangular number* t_n is defined by $t_n = 1 + 2 + \cdots + n$.

(a) Find a recurrence relation for s_n, where $n \geq 1$ and $s_n = t_1 + t_2 + \cdots + t_n$. (5%)

(b) Compute $a_0 + a_1 + a_2 + a_3 + \cdots$, where $s_n = a_0 + a_1 n + a_2 n^2 + a_3 n^3 + \cdots$. (5%)

13 Suppose $1 \leq a < b < c < d \leq 12$. How many sets $\{a, b, c, d\}$ are there, where no consecutive integers (e.g., 1 and 2, 2 and 3, 3 and 4, ...) appear in $\{a, b, c, d\}$? (10%)

14 A graph $G = (V, E)$ is bipartite, if the vertex set V can be partitioned into two subsets V_1, V_2 such that each edge in E connects a vertex in V_1 with a vertex in V_2. Further, a bipartite graph G is complete, if it has a maximal number, i.e., $|V_1| \times |V_2|$, of edges. Usually, a complete bipartite graph G is denoted by $K_{m,n}$, where $m = |V_1|$ and $n = |V_2|$.

(a) Suppose $m \times n = 16$ and $m \leq n$. Find the values of m, n such that $K_{m,n}$ has one or more Euler circuits, but has no Hamilton cycle? (5%)

(b) Generalize the result of (a), i.e., give conditions of m, n under which $K_{m,n}$ has one or more Euler circuits, but has no Hamilton cycle? (5%)

15 Suppose that $(R, +, \cdot)$ is a ring and S is a nonempty subset of R. Then, $(S, +, \cdot)$ is a ring if and only if

- for all $a, b \in S$, $a + b \in S$ and $a \cdot b \in S$;
- for all $a \in S$, $-a \in S$.

Please show that when S is finite, $(S, +, \cdot)$ is a ring if and only if for all $a, b \in S$, $a + b \in S$ and $a \cdot b \in S$. (10%)

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