



You need to define and explain your notations clearly. Points are granted only if detail computations are presented.

1. Consider the following problem:

$$\begin{aligned}
 \max \quad & z = x_1 + x_2 - 2x_3 \\
 \text{s.t.} \quad & x_1 + x_2 + 2x_3 \leq 100 \\
 & x_1 - 2x_2 \geq 10 \\
 & \frac{x_3}{x_1 + x_2} \geq \frac{1}{2} \\
 & x_1, x_2, x_3 \geq 0.
 \end{aligned}$$

- (a) Solve the problem by Simplex algorithm. (15%)
 (b) What are the reduced costs and shadow prices of the problem at optimality. (10%)
2. YunTech Transport is to deliver products from three factories to three sales centers. Information about the transportation is given below. A per unit shortage cost \$2 incurs for any unsatisfied demand, and a per unit holding cost \$1 incurs for any inventory.

Table 1: Information about the transportation

(a) Amounts of supply and demand				(b) Unit delivery cost				
	Supply		Demand	Center				
Factory 1	100	Center 1	130					
Factory 2	150	Center 2	100					
Factory 3	120	Center 3	80					
		Center 4	120					
				1	2	3	4	
				Factory 1	3	2	5	4
				Factory 2	9	10	8	20
				Factory 3	5	6	4	10

- (a) Formulate YunTech Transport's problem as a linear programming model. (10%)
 (b) Solve the problem (clearly show the basic variables and the objective value at optimality) (15%)
3. Consider a car repair shop with 2 repairmen and 2 waiting spaces for cars (not including spaces for repairing cars). The arriving failed car process is a Poisson process with rate 2 per hour. The car repair time is exponential with mean time of



20 minutes. The arriving car that can not find the a space to wait leave the shop immediately.

- (a) Find the expected number of waiting cars.(10%)
 - (b) Find the proportion of cars that leave repair shop without being repaired.(10%)
 - (c) If the average revenue generated by repairing a car is \$100, and cost of a repairman while repairing car is \$50 per hour, find the average net profit per hour of the car repair shop in steady state. (10%)
4. Assume that the yield of planting apple in a year is categorized into three types: good (G), average (A) and bad (B). When the yield is good, the revenue generated from apple is \$8000. When the yield is average, the revenue generated from apple is \$12000. When the yield is bad, the revenue generated from apple is \$6000. Assume that yield of apple this year depends on yield of apple last year. Let X_n be the yield of apple in year n. The changes of yield in each year have the following transition probabilities : $P(X_{n+1} = G|X_n = A) = P_{AG} = 0.2$, following with similar notation definition, $P_{AA} = 0.3$, $P_{AB} = 0.5$, $P_{GG} = 0.5$, $P_{GA} = 0.4$, $P_{GB} = 0.1$, $P_{BG} = 0.2$, $P_{BA} = 0.3$, $P_{BB} = 0.5$. Find the average revenue per year of apple in steady state. (20%)