國立新竹教育大學 100 學年度碩、博士班招生考試試題

所別：應用數學系碩士班

科目：微積分(本科總分 150 分，含初等微積分、高等微積分)

※ 請橫書作答

1. Find the limits $(a)\lim_{x\to 0}\left(\frac{1}{\sin x}-\frac{1}{x}\right)$ (7%) $(b)\lim_{x\to\infty}\left(1+\frac{2}{x^2}\right)^x$ (8%)

   $(c)\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{8}{n}\left[-1+\frac{4k}{n}\right]^4-\frac{32}{n}\right)$ (10%)

2. (a) State the Intermediate Value Theorem。(5%)

   (b) Show that there are always two points opposite from each other with the same temperature on a circular wire ring。(15%)

3. Suppose $f(x)=x^{\frac{1}{x^5}}$ prove that f is uniformly continuous on $[1,\infty)$。(20%)

4. Give an example of $\{a_{ik}\}$ and $\{b_i\}$ such that $a_{ik}>0$ for $i,k\in\mathbb{N}$, $b_i=\sum_{k=1}^{\infty}a_{ik}$ and $\sum_{i=1}^{\infty}b_i<\infty$。(10%)

5. Let $f:\mathbb{R}\to\mathbb{R}$ satisfy $|f(x)-f(y)|\le|x-y|^2$ for any $x,y\in\mathbb{R}$. Prove that f is a constant function. (15%)

6. Consider the function $F(x,y)=3x^4-4x^2y+y^2$.

   (a) Show that along every line $L\subset\mathbb{R}^2$ through the origin the restriction of F to L has a local minimum at $(0,0)$. (10%)

   (b) Prove or disprove that $F:\mathbb{R}^2\to\mathbb{R}$ has a local minimum at $(0,0)$. (15%)

7. Suppose $\{p_n\}$ and $\{q_n\}$ are Cauchy sequences in a metric space X. Show that the sequence $\{d(p_n,q_n)\}$ converges. (15%)

8. Consider the function $f(x)=\dfrac{\ln x}{x}$.

(a) Show that f is increasing on $(0,e)$ and decreasing on (e,∞). (10%)

(b) Explain which the following holds :

(i) $e^{\pi}>\pi^{e}$ (ii) $e^{\pi}<\pi^{e}$ (iii) $e^{\pi}=\pi^{e}$ (10%)