

# 逢甲大學107學年度碩士班考試入學試題

編號：07 科目代碼：205

科目	微積分	適用系所	統計學系統計與精算碩士班應用統計暨計量財務組、精算組	時間	90分鐘
----	-----	------	----------------------------	----	------

※請務必在答案卷作答區內作答。

共 1 頁 第 1 頁

- 1) (10%) If  $a > 0$  and  $b > 0$ , compute the limit,  $\lim_{x \rightarrow 0} \frac{(a^x - b^x)}{x}$ .
- 2) (10%) Define  $f(x)$  by  $f(x) = \begin{cases} x, & x \text{ rational} \\ 0, & x \text{ irrational} \end{cases}$ . Show that  $f$  is continuous at only one point and is differentiable there.
- 3) (10%) Let the function  $f$  be given by  $f(x) = \begin{cases} x^2, & \text{for } x \leq 2 \\ ax + b, & \text{for } x > 2 \end{cases}$ . What must be values of  $a$  and  $b$  in order for  $f(x)$  to have a continuous derivative?
- 4) (10%+10%) Differentiate (a)  $\sqrt{x^2 + 3x + 4}$  and (b)  $\left(\frac{x+1}{x-1}\right)^5$ .
- 5) Evaluate the following integrals
- (a) (10%)  $\int_1^2 (x^{107} + x^{-2} + 3) dx$
- (b) (5%)  $\int_{-1}^2 |y^3 + y^2| dy$
- (c) (5%)  $\int_0^1 (1 - x)^3 dx$
- (d) (5%)  $\int_3^{10} x \sqrt{x+6} dx$
- 6) (7%) Let  $f(x, y) = x^2y$ . Find  $\iint_A f(x, y) dx dy$ , where  $A = \{(x, y) \in \mathbb{R}^2 | 0 < x < y < 3x < 1\}$ .
- 7) (7%) Find the derivative of the function  $g(x) = \int_0^x \sqrt{t^2 + 1} dt$ . That is, find  $g'(x)$ . Also, state the reason/theorem you used.
- 8) (a) (5%) Evaluate  $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{i^4}{n^5}$ .
- (b) (6%) Determine whether the series  $\sum_{n=2}^{\infty} \frac{1}{n \cdot (\ln n)}$  is convergent or divergent. Give reasons.