

中原大學 107 學年度碩士班考試入學

107/3/7 10:10 AM~11:40 AM

應用數學系統計組

誠實是我們珍視的美德，
我們喜愛「拒絕作弊，堅守正直」的你！

科目：統計(分「統計(A)」及「統計(B)」兩部份計分，各佔 50 分) (共 1 頁，第 1 頁)

可使用計算機(僅限於四則運算、三角函數及對數等基本功能，可程式之功能不可使用)

不可使用計算機

----- (不可直接作答於試題，請作答於答案卷) -----

一、統計(A)，共 50 分

1. Let $P(A) = P(B) = P(C) = 1$, find $P(A \cap B \cap C)$. (10%)
2. Suppose that $X \sim N(u, \sigma^2)$. Show that $\left(\frac{X-u}{\sigma}\right)^2 \sim \chi^2(1)$. (10%)
3. Let X and Y be random variables, and $E(e^{t(X+Y)}) = E(e^{tX}) \cdot E(e^{tY})$. Are X and Y independent? Why? (10%)
4. Let $\theta \sim U[0, 2\pi]$. Find the p.d.f. of $\sin 2\theta$. (10%)
5. Let A and B be independent events with $P(A) = 0.7$ and $P(B) = 0.2$. Find $P(A \cup B)$. (10%)

二、統計(B)，共 50 分

1. Let X_1, X_2 be i.i.d. Uniform[0,1]. Find the p.d.f. of $X_1 + X_2$. (10%)
2. Let $X_1, \dots, X_n \sim N(\mu, \sigma^2)$ be i.i.d., where μ is restricted to $\mu \leq a$ or $\mu \geq b$ for some numbers $a < b$. Assume that σ^2 is known. Hence, the parameter space is $\Theta = (-\infty, a] \cup [b, \infty)$. Find M.L.E. $\hat{\mu}$ (with some figures to explain it). (20%)
3. Let $X_1, X_2 \sim N(0,1)$ be i.i.d..
 - (a) Find the p.d.f. of $\frac{X_1^2}{X_1^2 + X_2^2}$. (10%)
 - (b) Find $P(\sqrt{X_1^2 + X_2^2} < 2|X_1|)$. (10%)