

元智大學 107 學年度 碩士班 招生試題卷

系(所)別：工業工程與管理 組別：不分組
學系碩士班

科目：微積分

用紙第 / 頁共 / 頁

●不可使用電子計算機

1. (10 points) evaluate the integrals

a. $\int \frac{dx}{\sqrt{4x^2 + 4x + 3}}$

b. $\int \tan^4 x \sec^6 x \, dx$

2. Prove whether the following series coverage or diverge, show your work.

a. (5 points) $\sum_{n=1}^{\infty} \left(\frac{3}{2n+1} \right)$

b. (5 points) $\sum_{n=1}^{\infty} \frac{\cos(n)}{n}$

c. (10 points each) find the radius of convergence and interval of convergence of

$$\sum_{n=1}^{\infty} \frac{x^n}{5^n}$$

d. (10 points) Find the number of terms is needed for the error of estimated sum

of $\sum_{n=1}^{\infty} \left(\frac{5}{2n^5} \right)$ is less than 0.00005.

3. (15 points) Find the gradient and Hessian for the following functions

$$f(x, y) = 2x^2y + 2x^2 - 6y + 2xy$$

4. (15 points) $\iint_R xy \, dA$ where R is the region in the first quarter and bounded by

$$y=x^2 \text{ and } y=3x. \text{ Evaluate the integral.}$$

5. (10 points) calculate the iterated integral $\int_0^1 \int_0^1 xy \sqrt{x^2 + y^2} \, dx \, dy$.

6. (10 points) Show $\lim_{x \rightarrow 0} \frac{2 \sin(x)}{x} = 2$.

Good Luck!