(107)輔仁大學碩士班招生考試試題	考試日期：107 年 3 月 9 日第 2 節
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科目：微積分　　系所組：數學系乙組

1. (15 points) Let $f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0, \\ 0, & x = 0. \end{cases}$

   (a) (5 points) Find $\lim_{x \to 0} f(x)$. Give reasons for your answer.

   (b) (10 points) Prove or disprove that the function $f(x)$ is differentiable at $x = 0$.

2. (10 points) The region enclosed by the x-axis and the parabola $y = 4x - x^2$ is revolved about the vertical line $x = -2$ to generate a solid. Find the volume of the solid.

3. (20 points) Evaluate each of the following limits.

   (a) (10 points) $\lim_{x \to 0^+} x^x$.

   (b) (10 points) $\lim_{n \to \infty} \frac{1}{n^{16}} \left(1^{15} + 2^{15} + 3^{15} + \cdots + n^{15}\right)$.

4. (10 points) Evaluate $\int_0^{2\pi} x |\sin x| \, dx$.

5. (10 points) Find the plane tangent to the surface $z = 4x^2 + 2y^2$ at $(1, 1, 6)$.

6. (10 points) Investigate the convergence of the following series

   (a) (5 points) $\sum_{n=1}^{\infty} \frac{n^5}{3^n}$.

   (b) (5 points) $\sum_{n=1}^{\infty} \cos\left(\frac{1}{n}\right)$.

7. (10 points) Evaluate

$$\frac{d}{dx} \left(\int_{1/x}^{x^2} \frac{1}{t} \, dt \right).$$

8. (15 points) Find two numbers a and b with $a \leq b$ such that

$$\int_a^b (6 - x - x^2) \, dx$$

has its largest value.

※ 注意：1.考生須在「彌封答案卷」上作答。
2.本試題紙空白部份可當稿紙使用。
3.考生於作答時可否使用計算機、法典、字典或其他資料或工具，以簡章之規定為準。