

國立彰化師範大學 100 學年度碩士班招生考試試題

系所：車輛科技研究所

科目：工程數學

☆☆請在答案紙上作答☆☆

共 1 頁，第 1 頁

共佔 100 分每題配分置於題目後面

1. Find the Determine of the following matrix. (10%)

$$\begin{bmatrix} 2 & 0 & -4 & 6 \\ 4 & 5 & 1 & 0 \\ 0 & 2 & 6 & -1 \\ -3 & 8 & 9 & 1 \end{bmatrix}$$

2. Given $\mathbf{A} = \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$, Find $e^{\mathbf{A}}$. (15%)

3. Given $y'' + 0.2y' + 0.26y = 1.22e^{0.5x}$, $y(0) = 3.5$, $y'(0) = 0.35$, Find y : (10%)

4. Solve the initial value problem (10%)

$$y'' - y = 10\delta\left(t - \frac{1}{2}\right) - 100\delta(t - 1), \quad y(0) = 10, \quad y'(0) = 1$$

5. $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2} - \beta u$, where β is a constant. (15%)

$$u_x(0, t) = u_x(\ell, t) = 0, \quad u(x, 0) = 1. \text{ Find } u:$$

6. Evaluate (counterclockwise) (10%)

(a) $\oint_C \frac{30z^2 - 23z + 5}{(2z - 1)^2(3z - 1)} dz$, $C: |z| = 1$

(b) $\int_{-\infty}^{\infty} \frac{1}{x^2 + 4} dx$

7. Solve the given initial value problem (15%)

$$\begin{cases} y_1' = -y_1 + y_2 \\ y_2' = -y_1 - y_2 \end{cases}, \quad y_1(0) = 1, \quad y_2(0) = 0$$

8. Given the function $f(x) = \frac{x^2}{2}$, $-\pi < x < \pi$ (15%)

(a) Find its Fourier series (10%)

(b) Show that $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots = \frac{\pi^2}{6}$ (5%)