1. (20%) Find $f(t)$ if the Laplace Transforms $\mathcal{L}(f)$ equals :

   (a) $\dfrac{4s - 3\pi}{s^2 + \pi^2}$　　　　　　　　(b) $\dfrac{e^{-2\pi s} - e^{-8\pi s}}{s^2 + 1}$

2. (20%) Find the eigenvalues and eigenvectors: $\begin{bmatrix} 4 & 0 \\ 2 & -4 \end{bmatrix}$.

3. (10%) Integrate $\displaystyle\oint_C \dfrac{7z - 6}{z^2 - z} dz$, C : counterclockwise around the unit circle.

4. (20%) Euler equation is seen in the form of $x^2 y'' + Axy' + By = 0$, where A and B are constants. Let $t = \ln x$,

   (1) Prove that the Euler equation can be rewritten as $Y''(t) + (A-1)Y'(t) + BY(t) = 0$,
   (2) Please solve the equation $x^2 y'' + 3xy' - 8y = 0$.
   [Hint: let $Y(t) = y(e^t) = y(x)$].

5. (10%) Evaluate $\displaystyle\int_C (x^2 dx - y dz)$, with C the curve given by $x = 2t, y = t^2, z = -t; 1 \le t \le 2$.

6. (10%) Evaluate $\displaystyle\oint_C 3x\cos(2y)dx - 3x^2\sin(2y)dy$, with C any positively oriented closed path in the x-y plane.

7. (10%) Obtain the Laplacian of $u(x, y, z) = x^2 y^3 z^4$.