國立高雄大學 107 學年度研究所碩士班招生考試試題

科目：統計學 系所：金融管理學系 是否使用計算機：是
考試時間：100 分鐘 本科原始成績：100 分

1. (7%)

The following hypothesis test is to be conducted.
H_0: Median ≤ 150
H_1: Median > 150
A sample of 30 provided 22 observations greater than 150, 3 observations equal to 150, and 5 observations less than 150. Use α = 0.01. What is your conclusion? There is no score without the calculation process.

2.
Market betas for individual stocks are determined by simple linear regression. For each stock, the dependent variable is its quarterly percentage return (capital appreciation plus dividends) minus the percentage return that could be obtained from a risk-free investment (the Treasury Bill rate is used as the risk-free rate). The independent variable is the quarterly percentage return (capital appreciation plus dividends) for the stock market (S&P 500) minus the percentage return from a risk-free investment. An estimated regression equation is developed with quarterly data; the market beta for the stock is the slope of the estimated regression equation (b_1). The value of the market beta is often interpreted as a measure of the risk associated with the stock. Market betas greater than 1 indicate that the stock is more volatile than the market average; market betas less than 1 indicate that the stock is less volatile than the market average. Suppose that the following figures are the differences between the percentage return and the risk-free return for 10 quarters for the S&P 500 and Horizon Technology.

S&P 500 (%)	Horizon Technology (%)
1.2	-0.7
-2.5	-2.0
-3.0	-5.5
2.0	4.7
5.0	1.8
1.2	4.1
3.0	2.6
-1.0	2.0
0.5	-1.3
2.5	5.5

(1) (8%) Develop an estimated regression equation. What is Horizon Technology's market beta?

(2) (7%) Test for a significant relationship at the 0.05 level of significance.

(3) (7%) Did the estimated regression equation provide a good fit? Explain by using coefficient of determination.

3. (7%) The Wall Street Journal's Shareholder Scoreboard tracks the performance of 1000 major U.S. companies. The performance of each company is rated based on the annual total return, including stock price changes and the re-investment of dividends. Ratings are assigned by dividing all 1000 companies into five groups from A (top 20%), B (next 20%), to E (bottom 20%). Shown here are the one-year ratings for a sample of 60 of the largest companies. Do the largest companies differ in performance from the performance of the 1000 companies in the Shareholder Scoreboard? Use $\alpha = 0.05$. There is no score without the calculation process.

A	B	C	D	E
5	8	15	20	12

4. (7%) A local bank reviewed its credit card policy with the intention of recalling some of its credit cards. In the past approximately 5% of cardholders defaulted, leaving the bank unable to collect the outstanding balance. Hence, management established a prior probability of 0.05 that any particular cardholder will default. The bank also found that the probability of missing a monthly payment is 0.20 for customers who do not default. Of course, the probability of missing a monthly payment for those who default is 1. Given that a customer missed one or more monthly payments, compute the posterior probability that the customer will default. There is no score without the calculation process.

5. (7%) Suppose one has a stereo system consisting of two main parts, a radio and a speaker. If the lifetime of the radio is exponential with mean 1000 hours and the lifetime e of the speaker is exponential with mean 500 hours independent of the radio's lifetime, then what is the probability that the system's failure (when it occurs) will be caused by the radio failing?

6. (5%) Let $\mathrm{E}\{X\}$ and $\mathrm{Var}(X)$ denote the expectation and variance of a random variable X, respectively. Please show that:

$$\text{Var}(X) = \text{E}\{X^2\} - (\text{E}\{X\})^2.$$

7. Let X be normally distributed with the mean 0 and the variance 1, as well as $Y = \mu + \sigma X$ and $Z = e^Y$, where μ and $\sigma > 0$ are constant. Please answer the following questions:
   (1) (5%) What is the distribution of Y?
   (2) (10%) Following (1), please prove it.
   (3) (10%) Please evaluate the expectation of Z.

8. Let $W(t)$, where $t \geq 0$, be a stochastic process which satisfies the following conditions:

   i. $W(0) = 0$.

   ii. Given $0 \leq s < t$, $W(t) - W(s)$ is normally distributed with the mean 0 and the variance $t - s$.

   iii. Given $0 \leq t_1 < t_2 < t_3$, $W(t_2) - W(t_1)$ and $W(t_3) - W(t_2)$ are independent.

   Please answer the following questions:

   (1) (10%) Given $0 \leq s < t$, please show that the covariance of $W(s)$ and $W(t)$ is s. (Hint: $W(t) = (W(t) - W(s)) + W(s)$)

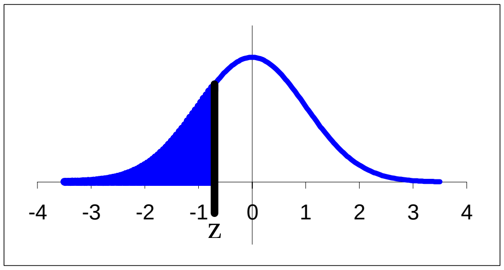
   (2) (10%) Let $\text{E}\{X|Y\}$ denote the expectation of a random variable X given another random variable Y. Given $0 \leq s < t$, please evaluate $\text{E}\{\text{E}\{W(t)|W(s)\}|W(0)\}$.

Table 1a: Standard Normal Probabilities

The values in the table below are cumulative probabilities for the standard normal distribution Z (that is, the normal distribution with mean 0 and standard deviation 1). These probabilities are values of the following integral:

$$P\left(Z \le z\right) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$$

Geometrically, the values represent the area to the left of z under the density curve of the standard normal distribution:



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Table 2: t-Distribution Critical Values

The entries in the table below are the critical values $t_{n,p}$, where n represents the number of degrees of freedom and p is the upper tail probability. That is, if T has the t-distribution with n degrees of freedom, then the value in the table represents the number $t_{n,p}$ such that $P(T > t_{n,p}) = p$.

	Upper Tail Probability p									
d.f.	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005
1	1.376	1.963	3.078	6.314	12.706	31.821	63.657	127.321	318.309	636.619
2	1.061	1.386	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.599
3	0.978	1.250	1.638	2.353	3.182	4.541	5.841	7.453	10.215	12.924
4	0.941	1.190	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.920	1.156	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.906	1.134	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.889	1.108	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.883	1.100	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.879	1.093	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.876	1.088	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.768
24	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
35	0.852	1.052	1.306	1.690	2.030	2.438	2.724	2.996	3.340	3.591
40	0.851	1.050	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
45	0.850	1.049	1.301	1.679	2.014	2.412	2.690	2.952	3.281	3.520
50	0.849	1.047	1.299	1.676	2.009	2.403	2.678	2.937	3.261	3.496
55	0.848	1.046	1.297	1.673	2.004	2.396	2.668	2.925	3.245	3.476
60	0.848	1.045	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
65	0.847	1.045	1.295	1.669	1.997	2.385	2.654	2.906	3.220	3.447
70	0.847	1.044	1.294	1.667	1.994	2.381	2.648	2.899	3.211	3.435
75	0.846	1.044	1.293	1.665	1.992	2.377	2.643	2.892	3.202	3.425
80	0.846	1.043	1.292	1.664	1.990	2.374	2.639	2.887	3.195	3.416
85	0.846	1.043	1.292	1.663	1.988	2.371	2.635	2.882	3.189	3.409
90	0.846	1.042	1.291	1.662	1.987	2.368	2.632	2.878	3.183	3.402
95	0.845	1.042	1.291	1.661	1.985	2.366	2.629	2.874	3.178	3.396
100	0.845	1.042	1.290	1.660	1.984	2.364	2.626	2.871	3.174	3.390
150	0.844	1.040	1.287	1.655	1.976	2.351	2.609	2.849	3.145	3.357
250	0.843	1.039	1.285	1.651	1.969	2.341	2.596	2.832	3.123	3.330
1000	0.842	1.037	1.282	1.646	1.962	2.330	2.581	2.813	3.098	3.300
∞	0.842	1.036	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Table 3: Chi-Square Distribution Critical Values

The entries in the table below are the critical values $\chi^2_{n,p}$, where n represents the number of degrees of freedom and p is the upper tail probability. That is, if X has the chi-square distribution with n degrees of freedom, then the value in the table represents the number $\chi^2_{n,p}$ such that $P\left(X > \chi^2_{n,p}\right) = p$.

	Upper Tail Probability p									
d.f.	0.995	0.975	0.95	0.90	0.80	0.20	0.10	0.05	0.025	0.005
1	0.000	0.001	0.004	0.016	0.064	1.642	2.706	3.841	5.024	7.879
2	0.010	0.051	0.103	0.211	0.446	3.219	4.605	5.991	7.378	10.597
3	0.072	0.216	0.352	0.584	1.005	4.642	6.251	7.815	9.348	12.838
4	0.207	0.484	0.711	1.064	1.649	5.989	7.779	9.488	11.143	14.860
5	0.412	0.831	1.145	1.610	2.343	7.289	9.236	11.070	12.833	16.750
6	0.676	1.237	1.635	2.204	3.070	8.558	10.645	12.592	14.449	18.548
7	0.989	1.690	2.167	2.833	3.822	9.803	12.017	14.067	16.013	20.278
8	1.344	2.180	2.733	3.490	4.594	11.030	13.362	15.507	17.535	21.955
9	1.735	2.700	3.325	4.168	5.380	12.242	14.684	16.919	19.023	23.589
10	2.156	3.247	3.940	4.865	6.179	13.442	15.987	18.307	20.483	25.188
11	2.603	3.816	4.575	5.578	6.989	14.631	17.275	19.675	21.920	26.757
12	3.074	4.404	5.226	6.304	7.807	15.812	18.549	21.026	23.337	28.300
13	3.565	5.009	5.892	7.042	8.634	16.985	19.812	22.362	24.736	29.819
14	4.075	5.629	6.571	7.790	9.467	18.151	21.064	23.685	26.119	31.319
15	4.601	6.262	7.261	8.547	10.307	19.311	22.307	24.996	27.488	32.801
16	5.142	6.908	7.962	9.312	11.152	20.465	23.542	26.296	28.845	34.267
17	5.697	7.564	8.672	10.085	12.002	21.615	24.769	27.587	30.191	35.718
18	6.265	8.231	9.390	10.865	12.857	22.760	25.989	28.869	31.526	37.156
19	6.844	8.907	10.117	11.651	13.716	23.900	27.204	30.144	32.852	38.582
20	7.434	9.591	10.851	12.443	14.578	25.038	28.412	31.410	34.170	39.997
21	8.034	10.283	11.591	13.240	15.445	26.171	29.615	32.671	35.479	41.401
22	8.643	10.982	12.338	14.041	16.314	27.301	30.813	33.924	36.781	42.796
23	9.260	11.689	13.091	14.848	17.187	28.429	32.007	35.172	38.076	44.181
24	9.886	12.401	13.848	15.659	18.062	29.553	33.196	36.415	39.364	45.559
25	10.520	13.120	14.611	16.473	18.940	30.675	34.382	37.652	40.646	46.928
26	11.160	13.844	15.379	17.292	19.820	31.795	35.563	38.885	41.923	48.290
27	11.808	14.573	16.151	18.114	20.703	32.912	36.741	40.113	43.195	49.645
28	12.461	15.308	16.928	18.939	21.588	34.027	37.916	41.337	44.461	50.993
29	13.121	16.047	17.708	19.768	22.475	35.139	39.087	42.557	45.722	52.336
30	13.787	16.791	18.493	20.599	23.364	36.250	40.256	43.773	46.979	53.672
31	14.458	17.539	19.281	21.434	24.255	37.359	41.422	44.985	48.232	55.003
32	15.134	18.291	20.072	22.271	25.148	38.466	42.585	46.194	49.480	56.328
33	15.815	19.047	20.867	23.110	26.042	39.572	43.745	47.400	50.725	57.648
34	16.501	19.806	21.664	23.952	26.938	40.676	44.903	48.602	51.966	58.964
35	17.192	20.569	22.465	24.797	27.836	41.778	46.059	49.802	53.203	60.275
36	17.887	21.336	23.269	25.643	28.735	42.879	47.212	50.998	54.437	61.581
37	18.586	22.106	24.075	26.492	29.635	43.978	48.363	52.192	55.668	62.883
38	19.289	22.878	24.884	27.343	30.537	45.076	49.513	53.384	56.896	64.181
39	19.996	23.654	25.695	28.196	31.441	46.173	50.660	54.572	58.120	65.476
40	20.707	24.433	26.509	29.051	32.345	47.269	51.805	55.758	59.342	66.766
45	24.311	28.366	30.612	33.350	36.884	52.729	57.505	61.656	65.410	73.166
50	27.991	32.357	34.764	37.689	41.449	58.164	63.167	67.505	71.420	79.490
60	35.534	40.482	43.188	46.459	50.641	68.972	74.397	79.082	83.298	91.952
70	43.275	48.758	51.739	55.329	59.898	79.715	85.527	90.531	95.023	104.215
80	51.172	57.153	60.391	64.278	69.207	90.405	96.578	101.879	106.629	116.321
90	59.196	65.647	69.126	73.291	78.558	101.054	107.565	113.145	118.136	128.299
100	67.328	74.222	77.929	82.358	87.945	111.667	118.498	124.342	129.561	140.169