

國立高雄大學 107 學年度研究所碩士班招生考試試題

科目：統計學  
考試時間：100 分鐘

系所：  
經營管理研究所(無組別)  
本科原始成績：100 分

是否使用計算機：是

I. (60%) MULTIPLE CHOICE QUESTION

所有的答案請寫在答案卷，答案寫法如下列所示：

All answers must be written on the answer sheet

for example:

1	a	6	a	11	d	16	a
2	b	7	b	12	d	17	a
3	c	8	c	13	b	18	c
4	d	9	d	14	c	19	c
5	a	10	a	15	d	20	c

1. Which of the following statements on a simple OLS regression function  $\hat{Y} = \hat{\alpha} + \hat{\beta}X$  is true ?

- a) If more samples are added to estimate the regression function, then the variance of  $\hat{\beta}$  will increase.
- b) Coefficient of determination  $R^2$  is equal to the correlation coefficient of the sample value  $Y$  and its fitted value  $\hat{Y}$ .
- c) For  $\hat{\beta}$  to be unbiased, the error terms should have the same variance for any given  $X$ .
- d) To get  $\hat{\beta}$ , the sample variance of  $X$  should be greater than zero.

2. Fourth moment of a normalized random variable is to measure the \_\_\_\_\_ of the distribution.

- a) kurtosis
- b) skewness
- c) symmetry
- d) biasness

3. Which of the following statements about normal distribution is **not** true ?

- a) It is a symmetric distribution.
- b) Linear combination of two independent normal distributed random variables with the same mean and variance is also a normal distribution.
- c) Standard normal distribution has zero mean and variance.
- d) Sum of the square of three standard normal distributions is a chi-square distribution.

4. Which of the following statements about binomial distribution is true?

- a) It is a symmetric distribution.
- b) Its mean and variance are equal.

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- c) Value of the random variable needs to be strictly positive.  
d) Binomial distribution sometimes asymptotically approaches Poisson distribution.
5. Which of the following statements about a  $F$  distribution is true?  
a) It is an asymmetric distribution.  
b) Value of the random variable can be negative.  
c) It is a ratio of two standardization normal distributions.  
d) The shape of its probability density curve is unrelated to the degree of freedoms.
6. Which of the following statements about  $p$ -value is true ?  
a) It is negatively related to the significant level.  
b) The lower the  $p$ -value the more likely it is to reject null hypothesis.  
c) It is positively related to the value of the parameter in null hypothesis.  
d)  $P$ -value is only used in  $t$ -test.
7. The covariance of two independent random variables  $X$  and  $Y$  is **not** equal to  
a) 0  
b)  $E(X - E(X))(Y - E(Y))$   
c)  $E[(X - E(X))Y]$   
d)  $E(XY)$
8. Which of the following distributions can be used to test whether the population means of two random variables are equal ?  
a)  $F$  distribution  
b) Chi-square distribution  
c) Poisson distribution  
d) Laplace distribution
9. The coefficient of determination  $R^2$  of an OLS simple regression function is equal to  
a) square of the correlation coefficient of explained variable and fitted value  
b) square of slope coefficient  
c) square of the correlation coefficient of explanatory variable and fitted value  
d) square of the correlation coefficient of explanatory variable and residual

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10. When we use  $n$  random samples  $(X_i, Y_i)$  to obtain the OLS regression function  $\hat{Y}_i = \hat{\alpha} + \hat{\beta}X_i$ , we already know the sample variance of  $X$  and  $Y$  are 400 and 160, respectively and the coefficient of determination  $R^2$  is 0.1, then the estimator  $\hat{\beta}$  is

- a) 0.2
- b) 0.25
- c) 0.3
- d) 0.4

11. In a simple OLS regression function, which of the following distribution can be used to test whether  $X$  has a significant impact on  $Y$ ?

- a) binomial distribution
- b) chi-square distribution
- c)  $F$  distribution
- d) Poisson distribution

12.  $P$ -value is the \_\_\_\_\_ significant level at which we could carry out the test and still fail to reject null hypothesis.

- a) smallest
- b) largest
- c) medium
- d) optimal

13. Which of the following distribution has equal mean and variance?

- a) binominal distribution
- b) Poisson distribution
- c) chi-square distribution
- d) exponential distribution.

14. If  $V(X) = 20$ ,  $V(Y) = 40$ ,  $COV(X, Y) = -10$ , then  $V\left(\frac{2X + 3Y}{4}\right) =$

- a) 10
- b) 20
- c) 27.5
- d) 35

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15. Let  $S$  denote the prison sentence in month, for people convicted of motorcycle theft in

Kaoshiung City. Suppose that the probability density function of  $S$  is given by  $f(s) = \frac{s^2}{72}$ ,

$0 \leq s \leq 6$ . The expected prison sentence is

- a) 3 months
- b) 3.5 months
- c) 4 months
- d) 4.5 months

16. If  $\bar{X}$  is an unbiased and consistent estimator of  $\mu$ , and  $Y$  is a random variable with zero mean and constant variance, then  $Z = \bar{X} + Y$  is a (an) \_\_\_\_\_ estimator of  $\mu$ .

- a) unbiased and consistent
- b) unbiased but inconsistent
- c) biased but consistent
- d) biased and inconsistent

17. Which of the following statements about the property of an estimator is true?

- a) Unbiased assures consistent.
- b) Consistent assures unbiased.
- c) The number of unbiased estimator is no more than that of consistent estimator.
- d) There may be more than one unbiased and consistent estimator.

18. If we use  $\sum X_i \hat{U}_i = 0$  to estimate the equation  $Y_i = \beta X_i + U_i$  where  $\hat{U}_i$  is the residual, and

$\hat{\beta}$  as the estimator of  $\beta$ , then

a)  $\hat{\beta}$  is a biased estimator of  $\beta$ .

b)  $\hat{\beta} = \frac{\sum X_i Y_i}{\sum (X_i - \bar{X})^2}$ .

c)  $\sum \hat{Y}_i \hat{U}_i = 0$  does not hold.

d) the regression line does not pass through  $(\bar{X}, \bar{Y})$ .

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19. Let  $Y_1, Y_2, Y_3, Y_4$  be independently samples from a population with mean  $\mu$  and variance  $\sigma^2$ . Which of the following estimators of  $\mu$  is most efficient than the others

- a)  $Y_1$
- b)  $\frac{Y_1 + Y_2}{2}$
- c)  $\frac{Y_1 + Y_2 + Y_3 + Y_4}{4}$
- d)  $\frac{1}{8}Y_1 + \frac{1}{4}Y_2 + \frac{1}{4}Y_3 + \frac{3}{8}Y_4$

20.  $\{X_1, X_2, \dots, X_n\}$  is independently sampled from a population with mean  $\mu$  and variance  $\sigma^2$ .

If  $\hat{\sigma}^2 = \frac{(X_1 - \mu)^2}{6} + \frac{(X_2 - \mu)^2}{3} + \frac{(X_3 - \mu)^2}{2}$  is an estimator of  $\sigma^2$ , then  $\hat{\sigma}^2$  is an \_\_\_\_\_ estimator of  $\sigma^2$ .

- a) unbiased and consistent
- b) unbiased but inconsistent
- c) biased but consistent
- d) biased and inconsistent

II. (20%) Assume that  $X$  has a linear impact on  $Y$  in the population as  $Y = \alpha + \beta X + \varepsilon$ . If we have 86 random samples of  $(X, Y)$ , and have  $S_{XY} = 16$ ,  $\bar{X} = 12$ ,  $\bar{Y} = 16$ ,  $S_X = 8$ ,  $S_Y = 5$ .

(a) What's the OLS regression function  $\hat{Y} = \hat{\alpha} + \hat{\beta}X$ ? The coefficient of determination  $R^2$ ?

Does  $X$  have significant impact on  $Y$ ? ( $\alpha = 0.05$ ).

hint:  $P(t > t_{84, \alpha=0.05} = 1.663) = 0.05$ 、 $P(t > F_{1,84, \alpha=0.05} = 3.95) = 0.05$ 、

$P(\chi > \chi_{84, \alpha=0.05} = 64.749) = 0.05$ .

(b) If we use the same samples but new variable  $Y^* = 100Y$ ,  $X^* = 10X$  to obtain the new OLS regression function, what's the new regression function?  $R^2$  of the new regression function?

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III. (10%) In II, if we use the same samples to estimate the new function  $Y = \alpha_1 + \beta_1(X_i - \bar{X}) + \varepsilon_1$ , then what's the OLS regression function?  $R^2$  ?

IV. (10%) Assume that the population equation is  $y = \beta_0 + \beta_1 x + u$ , and meets the first four Gauss-Markov assumptions,  $E(u|x) = 0$ ,  $Var(u|x) = \sigma^2$ . If we transform  $x_i$  to be

$z_i = \ln(2x_i^2 + 3)$ , and use  $\tilde{\beta}_1 = \frac{\sum_{i=1}^n (z_i - \bar{z})y_i}{\sum_{i=1}^n (z_i - \bar{z})x_i}$  to estimate  $\beta_1$ , try to find out the follows:

(a)  $E(\tilde{\beta}_1) = ?$

(b)  $Var(\tilde{\beta}_1) = ?$